HIMACHAL PRADESH BOARD OF SCHOOL EDUCATION, DHARAMSHALA

CLASS: XII

SUBJECT : MATHEMATICS (Full Syllabus)

TIME ALLOWED: 3 HOURS MAX.MARKS:80

Special Instructions:

i. While answering your Questions, you must indicate on your answer-book the same Question No. as appear in your Questions Paper.
ii. All Questions are compulsory.
iii. Internal choices have been provided in some questions. Attempt only one of the choices in such questions.
iv. Do not leave blank page / pages in your answer book.
v. Question numbers 1 – 16 are multiple choice questions (M.C.Q.) carrying 1 mark each.
vi. Question numbers 17 – 25 are of 3 marks each.
vii. Question numbers 26 – 28 are of 4 marks each.
viii. Question numbers 29 – 33 are of 5 marks each.
ix. Graph paper must be attached in between the answer book.

Q.1. Let f: R → R be defined as f(x) = 3x (1)
Choose the correct answer:

(a) f is one-one onto (b) f is many one onto
(c) f is one-one but not onto (d) f is neither one-one nor onto

Q.2. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to (1)

(a) $2\sqrt{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) 0

Q.3. If $\sin^{-1} x$ = y then: (1)

(a) $0 \leq y \leq \pi$ (b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Q.4. The number of all possible matrices of order 3×3 with each entry 0 or 1 is: (1)
(a) 27 (b) 18 (c) 81 (d) 512

Q.5. The second order derivative of $\log x$ is: (1)

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $-\frac{1}{x^2}$ (d) None of these

Q.6. The rate of change of the area of the circle with respect to its Radius r = 6 cm is: (1)

(a) 10π cm (b) 12π cm (c) 8πcm (d) 11πcm

Q.7. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3 % is: (1)

(a) 0.09 x^3m^3 (b) 0.9 x^3m^3 (c) 0.06 x^3m^3 (d) 0.6 x^3m^3

Q.8. $\int e^x s\quad (1 + t\quad)d$ equals to: (1)

(a) $e^x c\quad + C$ (b) $e^x s\quad + C$ (c) $e^x s\quad + C$ (d) $e^x t\quad + C$

Q.9. Area lying between the curves y^2 = 4x and y = 2x is: (1)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Q.10. The order of the differential equation (1)
$2x^2\frac{d^2y}{dx^2} - 3\frac{d}{d}$ + y = 0 is:

(a) 2 (b) 1 (c) 0 (d) not defined

Q.11. The integrating factor of the differential equation (1)
x$\frac{d}{d}$ – y =2x^2 is:

(a) e^{-x} (b) e^{-y} (c) x (d) $\frac{1}{x}$

Q.12. If $\vec{a}.\vec{b}$ =$|\vec{a}||\vec{b}|$, Then θ =? (1)

(a) $\frac{\pi}{4}$ (b) 0 (c) π (d) $\frac{\pi}{2}$

Q.13. Direction cosines of z- axis are: (1)

(a) 0, 1, 0 (b) 1, 0, 0 (c) 0, 0, 1 (d) None of these

Q.14. Distance of the plane 2x – y + 2z + 3 = 0 from the point (3, -2, 1) is: (1)

(a) $\frac{3}{1}$ (b) $\frac{1}{3}$ (c) 0 (d) 13

Q.15. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is: (1)

(a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{1}$ (d) 0

Q.16. If A and B are events such that P(A/B) = P(B/A),Then: (1)

(a) A⊂ B but A ≠ B (b) A = B (c) A∩B = ∅ (d) P(A) = P(B)

Q.17. Find gof and fog, if (3)

f(x) = $|x|$ and g(x) = $|5x - 2|$

Q.18. Using elementary transformations, find the inverse of (3)

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Q.19. Using the properties of the determinants, show that (3)

$$\begin{vmatrix} y + k & y & y \\ y & y + k & y \\ y & y & y + k \end{vmatrix} = k^2\ (3y + k)$$

Q.20. Find the value of k so that the function f defined by (3)

$$f(x) = \begin{cases} k\ \ + 1, i\ \ x \le \pi \\ \cos x, i\ \ x > \pi \end{cases}$$ is continuous at point x = π

Q.21. Evaluate $\int x\, l\ \ 2x\, d$ (3)

Q.22. By using the properties of the definite integrals, evaluate (3)

$$\int_0^{\frac{\pi}{2}} \frac{c\ \ ^5x}{s\ \ ^5x + c\ \ ^5x} d$$

Q.23. Solve the differential equation: (3)

$(x^2 - y^2)d\ \ + 2xy\ d\ \ = 0$

OR

Solve the differential equation:

$x\log x \frac{d}{d} + y = \frac{2}{x}\log x$

Q.24. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. (3)

Q.25. Find the probability distribution of number of tails in the simultaneous tosses of 3 coins. (3)

OR

Find the probability of getting 5 exactly twice in 7 throws of a die.

Q.26. Prove that (4)

$\tan^{-1}\frac{2}{1} + \tan^{-1}\frac{7}{2} = \tan^{-1}\frac{1}{2}$

OR

Express $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$ in the simplest form:

Q.27. Differentiate $\sin\{\tan^{-1}(e^x)\}$ with respect to x. (4)

OR

If $y^x = x^y$, find $\frac{d}{d}$

Q.28. Find the area of a parallelogram whose adjacent sides are determined by the vectors. (4)

$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$

Q.29. Solve the system of linear equations, using matrix method. (5)

x – y + z = 4
2x + y - 3z = 0
X + y + z = 2

Q.30. Find two positive numbers x and y such that x + y = 60 and xy^3 is maximum. (5)

OR

Find the equation of tangent and normal to the parabola

y^2 = 4ax at point (at^2, 2at)

Q.31. Find the area of the region bounded by the ellipse (5)

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

OR

Using integration find the area of region bounded by the triangle whose vertices are (-1,0), (1,3) and (3,2)

Q.32. Find the shortest distance between the lines (5)

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$
$\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

OR

Find the equation of plane through the intersection of the planes
$3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passes through the point (2, 2, 1).

Q.33. Solve the following linear programming problem (LPP) graphically:
Maximize $Z = 5x + 3y$ subject to the constraints (5)

$3x + 5y \leq 15$
$5x + 2y \leq 10$
$x \geq 0$
$y \geq 0$