Model Question Paper 2021-22
Mathematics
Class-12

TIME – 3 Hrs 15 Min   Maximum Marks - 100

Note: First 15 minutes are allotted for the candidates to read the question paper.

Instructions :

(i) There are in all *nine* questions in this question paper.

(ii) *All* questions are compulsory.

(iii) In the beginning of each question, the number of parts to be attempted has been clearly mentioned.

(iv) Marks allotted to the questions are indicated against them.

(v) Start solving from the first question and proceed to solve till the last one.

(vi) Do not waste your time over a question you cannot solve.

1. Choose the correct option and write down in your answer sheet.

(a) Suppose that the function defined as $f(x)=3x$ is $f: R \to R$, select the correct option. 01

(i) f is one-one onto (ii) f is many-one onto

(iii) f is one-one but not onto (iv) f is neither one-one nor onto

(b) If R is a relation on the set N, defined as R={(a,b): a=b-2, b>6}, select the correct option from the following. 01

(i) (2, 4) $\in R$ (ii) (3, 8) $\in R$

(iii) (6, 8) $\in R$ (iv) (8, 7) $\in R$

(c) Find the value of integral $\int xe^x dx$ 01

(i) e^x (ii) $(x+1)e^x$ (iii) $(x-1)e^x$ (iv) $\frac{x^2}{2}e^x$

(d) Order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ is - 01

(i) 2 (ii) 1 (iii) 0 (iv) not defined

(e) If the vector's $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-4\hat{j}+\lambda\hat{k}$ are mutually perpendicular, then find the value of λ - 01

(i) 3 (ii) 2 (iii) 4 (iv) 0

2. Attempt all the parts:

(a) Find the principal value of $\text{Cot}^{-1}\left(\frac{-1}{\sqrt{3}}\right)$. 01

(b) Show that the function $f(x)=|x|$, is continuous at $x=0$. 01

(c) Find the order and power of the differential equation $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$. 01

(d) Find the maximum value of $z=3x+4y$ subject to the following constraints $x+y\le 4, x\ge 0, y\ge 0$. 01

(e) If P(A)=$\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$ then find the value of $P(A/B)$. 01

3. Attempts all the parts:

(a) If $A=\{1,2\}$ and $B=\{3,4\}$ then find the number of relations between A and B. 02

(b) If $y=A\sin x+B\cos x$ then prove that $\frac{d^2y}{dx^2}+y=0$. 02

(c) Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$. 02

(d) A problem of mathematics is given to three students. Probabilities of solving the problem by them are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$. If all the three students try their best, then find the probability that problem is solved. 02

4. Attempt all the parts.

(a) Show that the function defined on R as $f(x) = 7x - 3$ is an increasing function. 02

(b) Find the unit vector perpendicular to each of vectors $(\bar{a}+\bar{b})$ and $(\bar{a}-\bar{b})$ where $\bar{a}=\hat{i}+\hat{j}+\hat{k}$, $\bar{b}=\hat{i}+2\hat{j}+3\hat{k}$. 02

(c) Find the area of parallelogram whose adjacent sides are given by vectors $\bar{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\bar{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$. 02

(d) A and B are two given events where $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{5}$ and $P(B)=P$. Find the value of P if events are mutually exclusive. 02

5. Attempt all the parts.

(a) Prove that the relation R on the set of integers Z is defined as R={(a, b) : (a-b) is divisible by number 2} is an equivalence relation. 05

(b) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$. 05

(c) Differentiate the function $(\sin x)^{\cos x}$ with respect to x. 05

(d) Find the $\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx$. 05

(e) Find the shortest distance between the lines $\bar{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\bar{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$. 05

6. Attempt all the parts:

(a) Show that the function $f(x) = \begin{cases} \frac{|x|}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$ is discontinuous at $x = 0$. 05

(b) Find the area bounded by the parabolas $y=x^2$ and $y^2=x$. 05

(c) Find the equation of the plane passing through the intersection of the planes $\bar{r}.(\hat{i}+\hat{j}+\hat{k})=6$ and $\bar{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$ and the point (1, 1, 1). 05

(d) Minimize $z = 3x + 2y$ subject to the constraints; 05

$x + y \geq 8, \quad 3x + 5y \leq 15, \quad x \geq 0, \quad y \geq 0$

(e) In a hostel 60% students read Hindi newspaper, 40% students read English newspaper and 20% read both newspapers -

(i) Find the probability of the students who read neither Hindi newspaper nor English newspaper. $2\frac{1}{2}$

(ii) If she reads Hindi newspaper then what is the probability that she also reads English newspaper. $2\frac{1}{2}$

7. Attempt any one of the following:

(a) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then find out the value of $(AB)^{-1}$. 08

(b) Solve the following system of linear equations by the matrix method: 08

$$\begin{aligned} 3x - 2y + 3z &= 8 \\ 2x + y - z &= 1 \\ 4x - 3y + 2z &= 4 \end{aligned}$$

8. Attempt any one of the following:

(a) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. 08

(b) Find the general solution of the differential equation $\frac{dy}{dx} - y = \text{Cos}x$. 08

9. Attempt any one of the following:

(a) Find the value of the integral $\int_0^{\frac{\pi}{2}} log sinx \, dx$. 08

(b) Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \text{Cos}^2 x + b^2 \text{Sin}^2 x}$. 08

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