

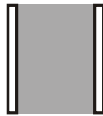
**FINAL JEE–MAIN EXAMINATION – JANUARY, 2020**

(Held On Tuesday 07<sup>th</sup> JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

**PHYSICS**

**TEST PAPER WITH ANSWER & SOLUTION**

1. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as  $k(x) = K(1 + \alpha x)$  where 'x' is the distance measured from one of the plates. If  $(\alpha d) \ll 1$ , the total capacitance of the system is best given by the expression :

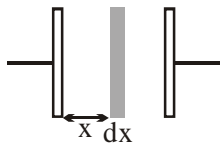


- (1)  $\frac{AK\epsilon_0}{d} \left(1 + \frac{\alpha d}{2}\right)$       (2)  $\frac{A\epsilon_0 K}{d} \left(1 + \left(\frac{\alpha d}{2}\right)^2\right)$   
 (3)  $\frac{A\epsilon_0 K}{d} \left(1 + \frac{\alpha^2 d^2}{2}\right)$       (4)  $\frac{AK\epsilon_0}{d} (1 + \alpha d)$

NTA Ans. (1)

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x  
 Capacitance of element

$$dC = \frac{(A)K(1 + \alpha x)\epsilon_0}{dx}$$



Now all such elements are in series so equivalent capacitance

$$\frac{1}{C} = \int \frac{1}{dC} = \int_0^d \frac{dx}{AK\epsilon_0(1 + \alpha x)}$$

$$\frac{1}{C} = \frac{1}{\alpha AK\epsilon_0} \ln\left(\frac{1 + \alpha d}{1}\right)$$

$$= \frac{1}{C} = \frac{1}{\alpha AK\epsilon_0} \left( \alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} + \dots \right)$$

$$\Rightarrow \frac{1}{C} = \frac{\alpha d}{\alpha AK\epsilon_0} \left( 1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} + \dots \right)$$

$$\frac{1}{C} = \frac{d}{AK\epsilon_0} \left( 1 - \frac{\alpha d}{2} \right)$$

$$C = \frac{AK\epsilon_0}{d} \left( 1 + \frac{\alpha d}{2} \right)$$

2. The time period of revolution of electron in its ground state orbit in a hydrogen atom is  $1.6 \times 10^{-16}$  s. The frequency of revolution of the electron in its first excited state (in  $s^{-1}$ ) is:  
 (1)  $6.2 \times 10^{15}$                       (2)  $5.6 \times 10^{12}$   
 (3)  $7.8 \times 10^{14}$                       (4)  $1.6 \times 10^{14}$

NTA Ans. (3)

Sol. Time period of revolution of electron in  $n^{\text{th}}$  orbit

$$T = \frac{2\pi r}{V} = \frac{2\pi a_0 \left(\frac{n^2}{Z}\right)}{V_0 \left(\frac{Z}{n}\right)}$$

$$\Rightarrow T \propto \frac{n^3}{Z^2}$$

$$\frac{T_2}{T_1} = \frac{(2)^3}{(1)^3} = 8 \Rightarrow T_2 = 8 \times 1.6 \times 10^{-16}$$

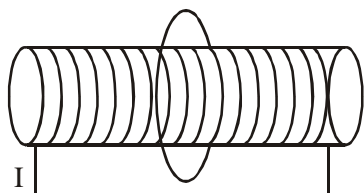
Now frequency  $f_2 = \frac{1}{T_2} = \frac{10^{16}}{8 \times 1.6} \approx 7.8 \times 10^{14}$  Hz.

3. A long solenoid of radius R carries a time (t)-dependent current  $I(t) = I_0(1 - t)$ . A ring of radius 2R is placed coaxially near its middle. During the time interval  $0 \leq t \leq 1$ , the induced current ( $I_R$ ) and the induced EMF ( $V_R$ ) in the ring change as :

- (1) At  $t = 0.5$  direction of  $I_R$  reverses and  $V_R$  is zero  
 (2) Direction of  $I_R$  remains unchanged and  $V_R$  is zero at  $t = 0.25$   
 (3) Direction of  $I_R$  remains unchanged and  $V_R$  is maximum at  $t = 0.5$   
 (4) At  $t = 0.25$  direction of  $I_R$  reverses and  $V_R$  is maximum

NTA Ans. (1)

Sol.



Magnetic flux ( $\phi$ ) through ring is  $\phi = \pi(R)^2 \cdot B$

$$\phi = (\pi R^2)(\mu_0 n I) = (\pi R^2 \mu_0 n I_0)(t - t^2)$$

Induced e.m.f. of  $V_R = \frac{-d\phi}{dt}$

$$= (\pi R^2 \mu_0 n I_0)(2t - 1)$$

and induced current  $I_R = \frac{\pi R^2 \mu_0 n I_0 (2t - 1)}{R_R}$

( $R_R \rightarrow$  Resistance of Ring)

Clearly  $V_R$  and  $I_R$  are zero at  $t = \frac{1}{2} = 0.5$  sec.

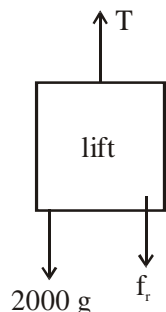
and their sign also changes at  $t = 0.5$  sec.

4. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to:

- (1 HP = 746 W,  $g = 10 \text{ ms}^{-2}$ )  
 (1)  $1.7 \text{ ms}^{-1}$                       (2)  $2.0 \text{ ms}^{-1}$   
 (3)  $1.9 \text{ ms}^{-1}$                       (4)  $1.5 \text{ ms}^{-1}$

NTA Ans. (3)

Sol.



Let elevator is moving upward with constant speed  $V$ .

Tension in cable

$$T = 2000 \text{ g} + f_r = 2000 + 4000$$

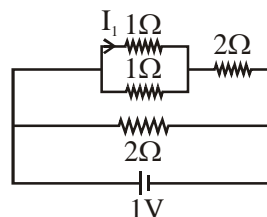
$$T = 24000 \text{ N}$$

$$\text{Power } P = TV$$

$$\Rightarrow 60 \times 746 = (24000) V$$

$$V = \frac{60 \times 746}{24000} = 1.865 \approx 1.9 \text{ m/s.}$$

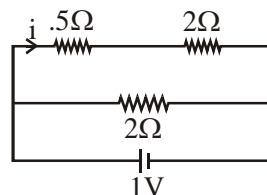
5. The current  $I_1$  (in A) flowing through  $1 \Omega$  resistor in the following circuit is :



- (1) 0.5                                      (2) 0.2  
 (3) 0.25                                   (4) 0.4

NTA Ans. (2)

Sol. Equivalent resistance of upper branch of circuit  $R = 2.5 \Omega$



Voltage across upper branch = 1 V

$$\Rightarrow i = \frac{1}{2.5} = .4 \text{ A}$$

$$\Rightarrow I_1 = 0.2 \text{ A}$$

6. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If  $\gamma = 1.40$ , the work done by air is : ( $3^{1.4} = 4.6555$ ) [Take air to be an ideal gas]

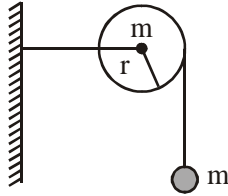
- (1) 90.5 J                                      (2) 48 J  
 (3) 60.7 J                                      (4) 100.8 J

NTA Ans. (1)

Sol.  $w = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1 V_1 - P_2 V_2}{0.4}$

$$= \frac{100 - \frac{100}{4.6555} \times 3}{0.4} = 88.90 \cdot$$

7. As shown in the figure, a bob of mass  $m$  is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius  $r$  and mass  $m$ . When released from rest the bob starts falling vertically. When it has covered a distance of  $h$ , the angular speed of the wheel will be :



- (1)  $\frac{1}{r}\sqrt{\frac{2gh}{3}}$  (2)  $r\sqrt{\frac{3}{4gh}}$  (3)  $\frac{1}{r}\sqrt{\frac{4gh}{3}}$  (4)  $r\sqrt{\frac{3}{2gh}}$

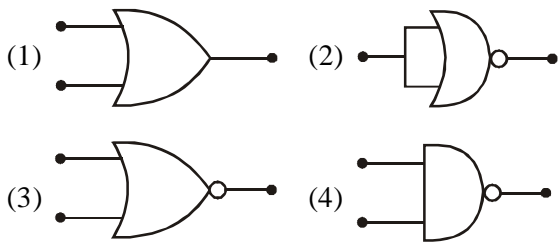
NTA Ans. (3)

Sol.  $mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2} = \frac{3}{4}mv^2$

$$u = \sqrt{\frac{4}{3}gh}$$

$$\omega = \frac{v}{r}$$

8. Which of the following gives a reversible operation?



NTA Ans. (2)

9. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to :

- (1) 22 mm (2) 12 mm  
(3) 33 mm (4) 2 mm

NTA Ans. (1)

Sol.  $m = \frac{LD}{f_e \times f_o} = \frac{150 \times 250}{f_e \times 25} = 375$

$$f_e = 20 \text{ mm.}$$

10. The radius of gyration of a uniform rod of length  $l$ , about an axis passing through a point  $\frac{l}{4}$  away from the centre of the rod, and perpendicular to it, is :

- (1)  $\frac{1}{8}l$  (2)  $\sqrt{\frac{7}{48}}l$  (3)  $\sqrt{\frac{3}{8}}l$  (4)  $\frac{1}{4}l$

NTA Ans. (2)

Sol.  $m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

$$\frac{7l^2}{48} = k^2$$

11. If the magnetic field in a plane electromagnetic wave is given by  $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{j} \text{ T}$ , then what will be expression for electric field?

- (1)  $\vec{E} = (9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V/m})$   
(2)  $\vec{E} = (3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{i} \text{ V/m})$   
(3)  $\vec{E} = (60 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V/m})$   
(4)  $\vec{E} = (3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{j} \text{ V/m})$

NTA Ans. (1)

Sol.  $\vec{E} \times \vec{B} = \vec{C} = -\hat{i}$

where  $\vec{B}$  is along  $\hat{j}$

$$\frac{E}{B} = C$$

$$E = 3 \times 10^{-8} \times 3 \times 10^8 = 9 \text{ V/m.}$$

12. Consider a circular coil of wire carrying constant current  $I$ , forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by  $\phi_i$ . The magnetic flux through the area of the circular coil area is given by  $\phi_0$ . Which of the following option is correct ?

- (1)  $\phi_i = -\phi_0$  (2)  $\phi_i = \phi_0$   
(3)  $\phi_i < \phi_0$  (4)  $\phi_i > \phi_0$

NTA Ans. (1)

13. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm<sup>2</sup>) is 90 ms<sup>-1</sup>. If the Young's modulus of wire is 16 × 10<sup>11</sup> Nm<sup>-2</sup>, the extension of wire over its natural length is :

- (1) 0.02 mm                      (2) 0.04 mm  
 (3) 0.03 mm                      (4) 0.01 mm

NTA Ans. (3)

Sol.  $v = \sqrt{\frac{T}{\mu}}$

$$90 = \sqrt{\frac{\frac{YA}{l} \Delta l}{\frac{m}{l}}} = \sqrt{\frac{16 \times 10^{11} \times 10^{-6} \times \Delta l}{6 \times 10^{-3}}}$$

$$= \frac{8100 \times 3}{8} \times 10^{-8} = \Delta l$$

14. Visible light of wavelength 6000 × 10<sup>-8</sup> cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ<sub>1</sub>, then θ<sub>1</sub> is close to :

- (1) 20°      (2) 45°      (3) 30°      (4) 25°

NTA Ans. (4)

Sol.  $\sin \theta = \frac{2\lambda}{\omega}$

$$\sin 60^\circ = \frac{2\lambda}{\omega}$$

$$\sin \theta_1 = \frac{\lambda}{\omega} = \frac{\sqrt{3}}{4}$$

$$\theta_1 = 25^\circ$$

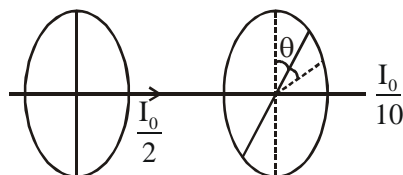
15. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is :

- (1) 18.4°      (2) 71.6°      (3) 90°      (4) 45°

NTA Ans. (1)

Sol.  $\frac{I_0}{10} = I = \frac{I_0}{2} \times \cos^2 \theta$

$$\cos \theta = \frac{1}{\sqrt{5}}$$



$$\theta = 63.44^\circ$$

$$\text{angle rotated} = 90 - 63.44^\circ = 26.56^\circ$$

Closest is 1.

16. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius

of the earth), it ejects a rocket of mass  $\frac{m}{10}$  so

that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

(1)  $\frac{m}{20} \left( u - \sqrt{\frac{2GM}{3R}} \right)^2$

(2)  $5m \left( u^2 - \frac{119 GM}{200 R} \right)$

(3)  $\frac{3m}{8} \left( u + \sqrt{\frac{5GM}{6R}} \right)^2$

(4)  $\frac{m}{20} \left( u^2 + \frac{113 GM}{200 R} \right)$

NTA Ans. (2)

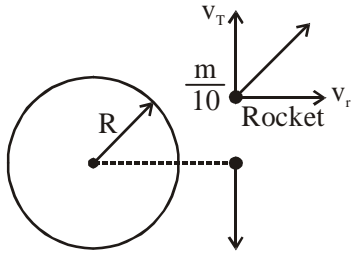
Sol. Applying energy conservation

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} mu^2 + \left( -\frac{GMm}{R} \right) = \frac{1}{2} mv^2 - \frac{GMm}{2R}$$

$$v = \sqrt{u^2 - \frac{GM}{R}} \quad \dots(i)$$

By momentum conservation, we have



$$\frac{m}{10} v_T = \frac{9m}{10} \sqrt{\frac{GM}{2R}} \quad \dots(ii)$$

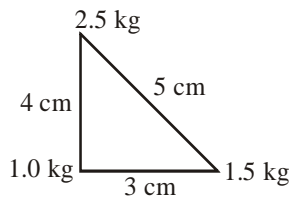
$$\& \frac{m}{10} v_r = mv$$

$$\Rightarrow \frac{m}{10} v_r = m \sqrt{u^2 - \frac{GM}{R}} \quad \dots(iii)$$

Kinetic energy of rocket

$$\begin{aligned} &= \frac{1}{2} m (v_T^2 + v_r^2) \\ &= \frac{m}{20} \left( 81 \frac{GM}{2R} + 100u^2 - 100 \frac{GM}{R} \right) \\ &= \frac{m}{20} \left( 100u^2 - \frac{119GM}{2R} \right) \\ &= 5m \left( u^2 - \frac{119GM}{200R} \right). \end{aligned}$$

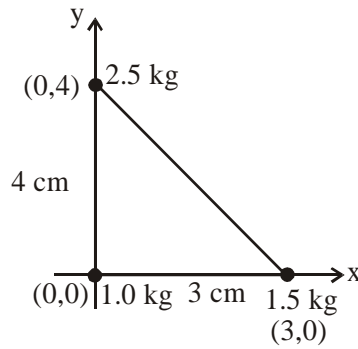
17. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



- (1) 1.5 cm right and 1.2 cm above 1 kg mass  
 (2) 0.9 cm right and 2.0 cm above 1 kg mass  
 (3) 0.6 cm right and 2.0 cm above 1 kg mass  
 (4) 2.0 cm right and 0.9 cm above 1 kg mass

NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

$$x_{cm} = \frac{1(0) + 1.5(3) + 2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0) + 1.5(0) + 2.5(4)}{5} = 2 \text{ cm}$$

18. Two moles of an ideal gas with  $\frac{C_P}{C_V} = \frac{5}{3}$  are mixed with 3 moles of another ideal gas with  $\frac{C_P}{C_V} = \frac{4}{3}$ . The value of  $\frac{C_P}{C_V}$  for the mixture is:

- (1) 1.50 (2) 1.42  
 (3) 1.45 (4) 1.47

NTA Ans. (2)

Sol. For 1<sup>st</sup> gas  $\frac{C_{P1}}{C_{V1}} = \frac{5}{3} \Rightarrow C_{P1} = 5x$  and  $C_{V1} = 3x$

For 2<sup>nd</sup> gas  $\frac{C_{P2}}{C_{V2}} = \frac{4}{3} \Rightarrow C_{P2} = 4x$  and  $C_{V2} = 3x$

Now for mixture  $C_P = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2} = \frac{17R}{5}$

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{12R}{5}$$

$$\Rightarrow \frac{C_P}{C_V} = \frac{2(5x) + 3(4x)}{2(3x) + 3(3x)} = \frac{17}{12}$$

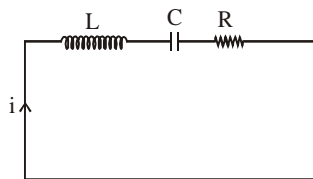
$$\Rightarrow \frac{C_P}{C_V} \approx 1.42.$$

19. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be:

- (1)  $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$
- (2)  $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$
- (3)  $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$
- (4)  $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

NTA Ans. (1)

Sol.



By kVL

$$-L \frac{di}{dt} - \frac{q}{C} - iR = 0$$

$$L \frac{d^2q}{dt^2} + \frac{1}{C}q + R \frac{dq}{dt} = 0$$

for damped oscillator

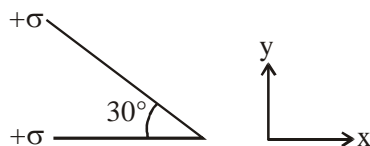
$$\text{net force} = -kx - bv = ma$$

$$\frac{md^2x}{dt^2} + kx + \frac{bdx}{dt} = 0$$

by comparing ; Equivalence is

$$L \rightarrow m ; C \rightarrow \frac{1}{K} ; R \rightarrow b.$$

20. Two infinite planes each with uniform surface charge density  $+\sigma$  are kept in such a way that the angle between them is  $30^\circ$ . The electric field in the region shown between them is given by:



$$(1) \frac{\sigma}{\epsilon_0} \left[ \left( 1 + \frac{\sqrt{3}}{2} \right) \hat{y} + \frac{\hat{x}}{2} \right]$$

$$(2) \frac{\sigma}{2\epsilon_0} \left[ \left( 1 - \frac{\sqrt{3}}{2} \right) \hat{y} - \frac{\hat{x}}{2} \right]$$

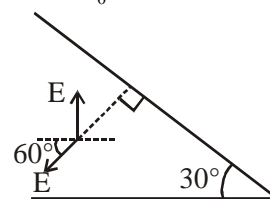
$$(3) \frac{\sigma}{2\epsilon_0} \left[ \left( 1 + \sqrt{3} \right) \hat{y} + \frac{\hat{x}}{2} \right]$$

$$(4) \frac{\sigma}{2\epsilon_0} \left[ \left( 1 + \sqrt{3} \right) \hat{y} - \frac{\hat{x}}{2} \right]$$

NTA Ans. (2)

Sol. Electric field due to each sheet is uniform and

$$\text{equal to } E = \frac{\sigma}{2\epsilon_0}$$

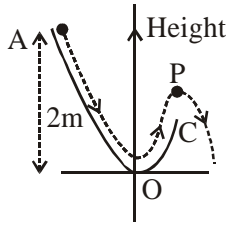


Now net electric field between plates

$$\vec{E}_{\text{net}} = E \cos 60^\circ (-\hat{x}) + (E - E \sin 60^\circ) (\hat{y})$$

$$= \frac{\sigma}{2\epsilon_0} \left[ -\frac{\hat{x}}{2} + \left( 1 - \frac{\sqrt{3}}{2} \right) \hat{y} \right].$$

21. A particle ( $m = 1 \text{ kg}$ ) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take  $g = 10 \text{ ms}^{-2}$ )\_\_\_\_\_.



NTA Ans. (10)

Sol. Mechanical energy conservation between A & P

$$U_1 + K_1 = K_2 + U_2$$

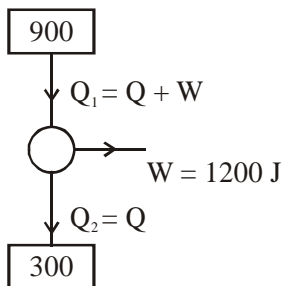
$$mg \times 2 = mg \times 1 + K_2$$

$$K_2 = mg \times 1 = 10 \text{ J.}$$

22. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is \_\_\_\_\_.

NTA Ans. (600)

Sol.



for carnot engine

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q+1200}{Q} = \frac{900}{300}$$

$$Q + 1200 = 3Q$$

$$Q = 600 \text{ J.}$$

23. A beam of electromagnetic radiation of intensity  $6.4 \times 10^{-5} \text{ W/cm}^2$  is comprised of wavelength,  $\lambda = 310 \text{ nm}$ . It falls normally on a metal (work function  $\phi = 2\text{eV}$ ) of surface area of  $1 \text{ cm}^2$ . If one in  $10^3$  photons ejects an electron, total number of electrons ejected in 1 s is  $10^x$ . ( $hc=1240 \text{ eVnm}$ ,  $1\text{eV}=1.6 \times 10^{-19} \text{ J}$ ), then x is \_\_\_\_\_.

NTA Ans. (10)

Sol. Power incident  $P = I \times A$

$n =$  no. of photons incident/second

$$nE_{\text{ph}} = IA$$

$$n = \frac{IA}{E_{\text{ph}}}$$

$$n = \frac{IA}{\left(\frac{hc}{\lambda}\right)} = \frac{6.4 \times 10^{-5} \times 1}{\frac{1240}{310} \times 1.6 \times 10^{-19}}$$

$$n = 10^{+14} \text{ per second}$$

Since efficiency =  $10^{-3}$

no. of electrons emitted =  $10^{+11}$  per second.

$$x = 11.$$

24. A non-isotropic solid metal cube has coefficients of linear expansion as :

$5 \times 10^{-5}/^\circ\text{C}$  along the x-axis and  $5 \times 10^{-6}/^\circ\text{C}$  along the y and the z-axis. If the coefficient of volume expansion of the solid is  $C \times 10^{-16}/^\circ\text{C}$  then the value of C is \_\_\_\_\_.

NTA Ans. (60)

**Sol.**  $\gamma = \alpha_x + \alpha_y + \alpha_z$

$$= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$$

$$= (50 + 5 + 5) \times 10^{-6}$$

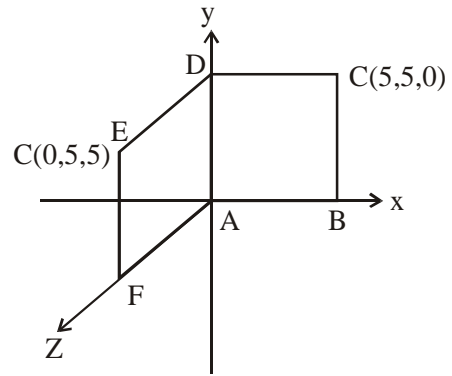
$$\gamma = 60 \times 10^{-6}$$

$$C = 60.$$

**25.** A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is  $\vec{B} = (3\hat{i} + 4\hat{k})T$ .

The quantity of flux through the loop ABCDEFA (in Wb) is \_\_\_\_\_ .

**NTA Ans. (175)**



**Sol.**

$$\vec{A}_{ABCD} = 25\hat{k}$$

$$\vec{A}_{ADEF} = 25\hat{i}$$

$$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$$

$$\vec{B} = 3\hat{i} + 4\hat{k}$$

$$\phi = \vec{B} \cdot \vec{A}$$

$$= 25 \times 3 + 25 \times 4$$

$$\phi = 175 \text{ W}_b.$$



**FINAL JEE-MAIN EXAMINATION – JANUARY, 2020**

(Held On Tuesday 07<sup>th</sup> JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

**CHEMISTRY**

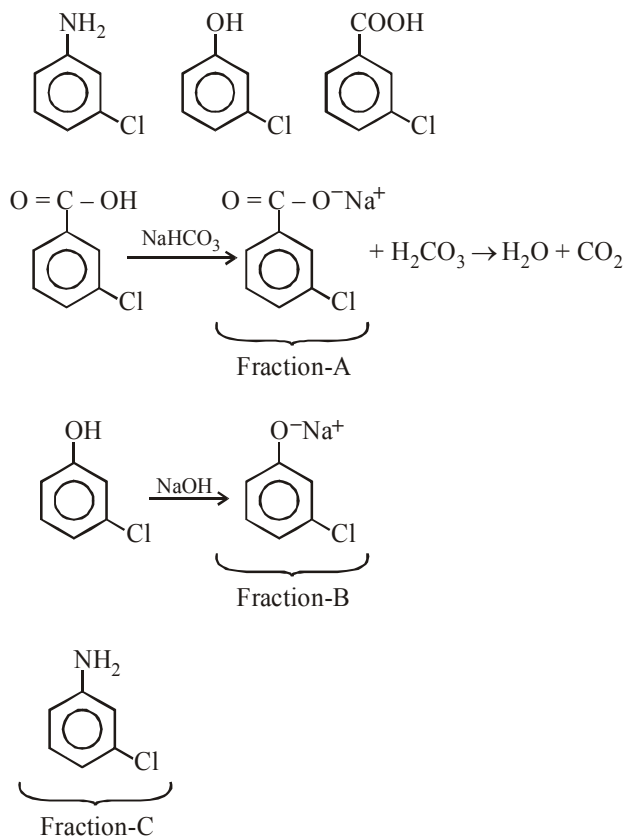
**TEST PAPER WITH ANSWER & SOLUTION**

1. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of  $\text{NaHCO}_3$  to give fraction A. The left over organic phase was extracted with dilute  $\text{NaOH}$  solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C, contain respectively :

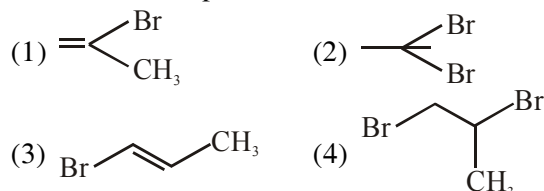
- (1) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol
- (2) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol
- (3) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
- (4) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline

NTA Ans. (3)

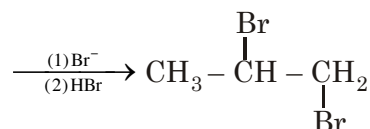
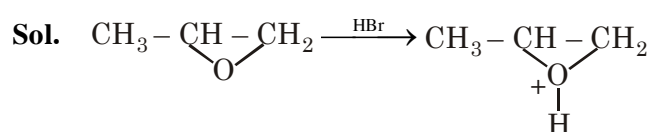
Sol.



2. 1-methyl ethylene oxide when treated with an excess of  $\text{HBr}$  produces :



NTA Ans. (4)



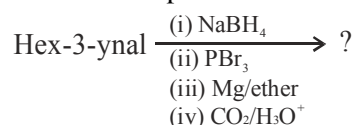
3. Amongst the following statements, that which was not proposed by Dalton was :

- (1) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.
- (2) chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction.
- (3) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.
- (4) matter consists of indivisible atoms.

NTA Ans. (3)

- Sol. Option(3) is according to Gaylussac's law of volume combination.

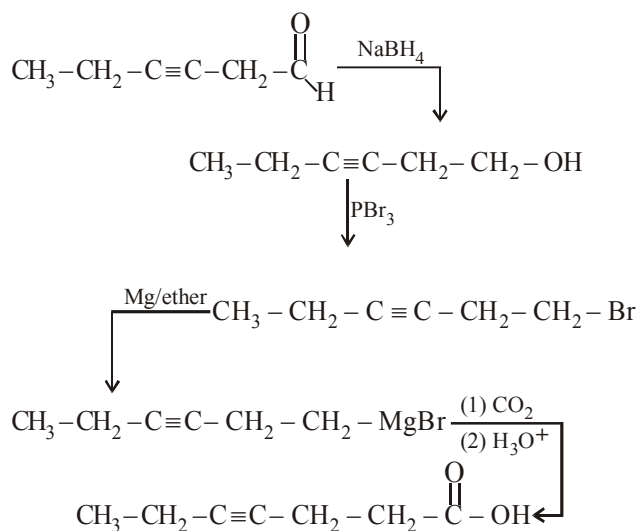
4. What is the product of following reaction ?



- (1)
- (2)
- (3)
- (4)

NTA Ans. (3)

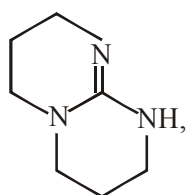
Sol.



5. The increasing order of  $pK_b$  for the following compounds will be :



(A)



(B)



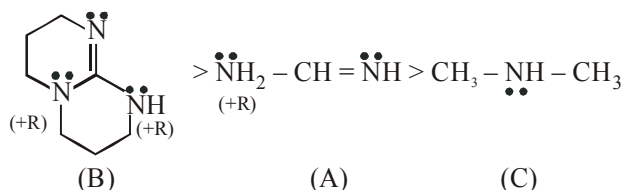
(C)

(1) (A) < (B) < (C)      (2) (C) < (A) < (B)

(3) (B) < (A) < (C)      (4) (B) < (C) < (A)

NTA Ans. (3)

Sol. Base strength order



increasing order of  $pK_b$  order (B < A < C)

6. The atomic radius of Ag is closest to :

(1) Cu      (2) Hg      (3) Au      (4) Ni

NTA Ans. (3)

Sol. Atomic radius of Ag and Au is nearly same due to lanthanide contraction.

7. The dipole moments of  $\text{CCl}_4$ ,  $\text{CHCl}_3$  and  $\text{CH}_4$  are in the order :

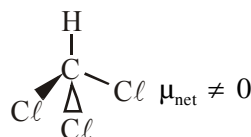
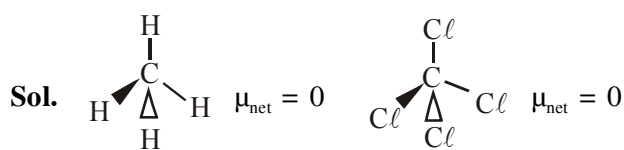
(1)  $\text{CH}_4 = \text{CCl}_4 < \text{CHCl}_3$

(2)  $\text{CH}_4 < \text{CCl}_4 < \text{CHCl}_3$

(3)  $\text{CCl}_4 < \text{CH}_4 < \text{CHCl}_3$

(4)  $\text{CHCl}_3 < \text{CH}_4 = \text{CCl}_4$

NTA Ans. (1)



8. Given that the standard potentials ( $E^\circ$ ) of  $\text{Cu}^{2+}/\text{Cu}$  and  $\text{Cu}^+/\text{Cu}$  are 0.34 V and 0.522 V respectively, the  $E^\circ$  of  $\text{Cu}^{2+}/\text{Cu}^+$  is :

(1) +0.158 V

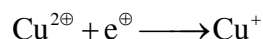
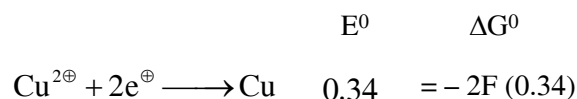
(2) 0.182 V

(3) -0.182 V

(4) -0.158 V

NTA Ans. (1)

Sol.



$$\Delta G^\circ = -2F(0.34) - (-F(0.522)) = -F(0.68 - 0.522) = -F(0.158)$$

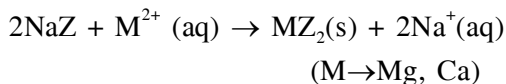
$$E^\circ = \frac{-F(0.158)}{-F} = 0.158\text{V}$$

9. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is :

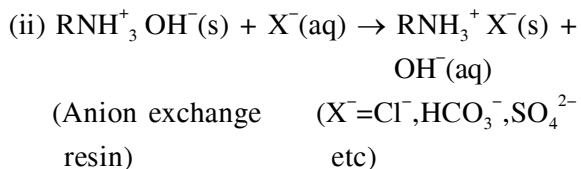
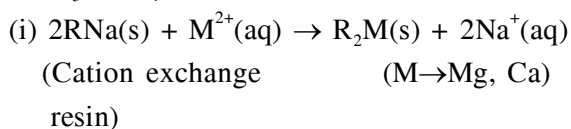
- (1) less efficient as it exchanges only anions
- (2) more efficient as it can exchange only cations
- (3) less efficient as the resins cannot be regenerated
- (4) more efficient as it can exchange both cations as well as anions

NTA Ans. (4)

Sol. (a) Zeolite method removes only cations ( $\text{Ca}^{2+}$  and  $\text{Mg}^{2+}$  ion) present in hard water



(b) Synthetic resin method removes cations ( $\text{Ca}^{2+}$  and  $\text{Mg}^{2+}$  ion) and anions (like  $\text{Cl}^-$ ,  $\text{HCO}_3^-$ ,  $\text{SO}_4^{2-}$  etc.)



10. The relative strength of interionic/intermolecular forces in decreasing order is :

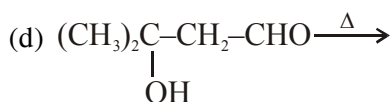
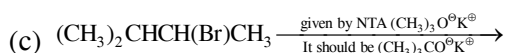
- (1) ion-dipole > ion-ion > dipole-dipole
- (2) dipole-dipole > ion-dipole > ion-ion
- (3) ion-dipole > dipole-dipole > ion-ion
- (4) ion-ion > ion-dipole > dipole-dipole

NTA Ans. (4)

Sol. Order is

ion – ion > ion – dipole > dipole – dipole

11. Consider the following reactions :

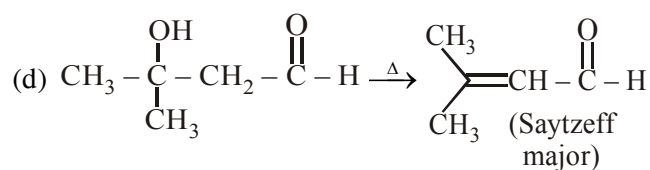
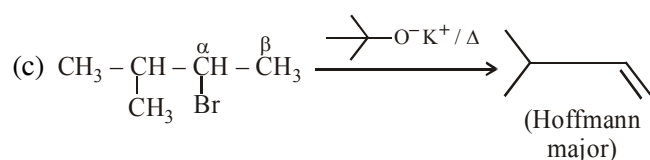
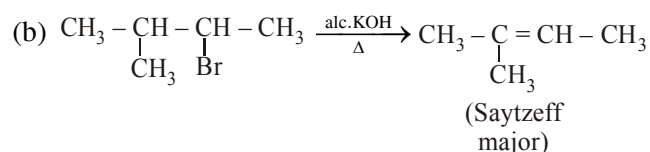
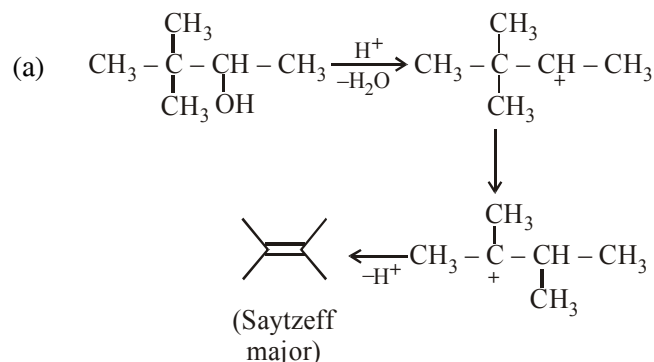


Which of these reaction(s) will not produce Saytzeff product ?

- (1) (c) only
- (2) (a), (c) and (d)
- (3) (d) only
- (4) (b) and (d)

NTA Ans. (1)

Sol.



$(\text{CH}_3)_3\text{O}^-\text{K}^+$  is incorrect representation of potassium tert-butoxide [ $(\text{CH}_3)_3\text{CO}^-\text{K}^+$ ].

So it is possible that it can be given as **Bonus**

12. The purest form of commercial iron is

- (1) scrap iron and pig iron
- (2) wrought iron
- (3) cast iron
- (4) pig iron

NTA Ans. (2)

**Sol.** Wrought iron is purest form of commercial iron.

**13.** At 35°C, the vapour pressure of CS<sub>2</sub> is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS<sub>2</sub> in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is :

- (1) heat must be absorbed in order to produce the solution at 35°C
- (2) Raoult's law is not obeyed by this system
- (3) a mixture of 100 mL CS<sub>2</sub> and 100 mL acetone has a volume < 200 mL
- (4) CS<sub>2</sub> and acetone are less attracted to each other than to themselves

**NTA Ans. (3)**

**Sol.** The vapour pressure of mixture (= 600 mm Hg) is greater than the individual vapour pressure of its constituents (Vapour pressure of CS<sub>2</sub> = 512 mm Hg, acetone = 344 mm Hg). Hence, the solution formed shows positive deviation from Raoult's law.

⇒ (1)  $\Delta_{\text{sol}}H > 0$  , (2) Raoult's law is not obeyed

(3)  $\Delta_{\text{sol. Volume}} > 0$

(4) CS<sub>2</sub> and Acetone are less attracted to each other than to themselves.

**14.** The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine and iodine, respectively are :

- (1) - 333, - 349, - 325 and - 296
- (2) -296, - 325, - 333 and - 349
- (3) - 333, - 325, - 349 and - 296
- (4) -349, - 333, - 325 and - 296

**NTA Ans. (1)**

**Sol.** Order of electron gain enthalpy (magnitude) is Cl > F > Br > I

**15.** The number of orbitals associated with quantum numbers  $n = 5$ ,  $m_s = +\frac{1}{2}$  is :

- (1) 11
- (2) 25
- (3) 15
- (4) 50

**NTA Ans. (2)**

**Sol.** No. of orbitals =  $n^2 = 5^2 = 25$

For  $n = 5$ , no. of orbitals =  $n^2 = 25$

Total number of orbitals is equal to no. of

electrons having  $m_s = \frac{1}{2}$

**16.** Match the following :

- |                    |                 |
|--------------------|-----------------|
| (i) Riboflavin     | (a) Beriberi    |
| (ii) Thiamine      | (b) Scurvy      |
| (iii) Pyridoxine   | (c) Cheilosis   |
| (iv) Ascorbic acid | (d) Convulsions |

(1) (i)-(c), (ii)-(a), (iii)-(d), (iv)-(b)

(2) (i)-(c), (ii)-(d), (iii)-(a), (iv)-(b)

(3) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(c)

(4) (i)-(a), (ii)-(d), (iii)-(c), (iv)-(b)

**NTA Ans. (1)**

**Sol.** (i) Riboflavin → (c) Cheilosis

(ii) Thiamine → (a) Beriberi

(iii) Pyridoxine → (d) Convulsions

(iv) Ascorbic acid → (b) Scurvy

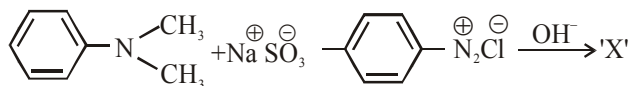
**17.** The theory that can completely/properly explain the nature of bonding in [Ni(CO)<sub>4</sub>] is :

- (1) Werner's theory
- (2) Crystal field theory
- (3) Valence bond theory
- (4) Molecular orbital theory

**NTA Ans. (4)**

**Sol.** In complex  $[\text{Ni}(\text{CO})_4]$  decrease in Ni-C bond length and increase in C-O bond length as well as its magnetic property is explained by MOT.

**18.** Consider the following reaction :

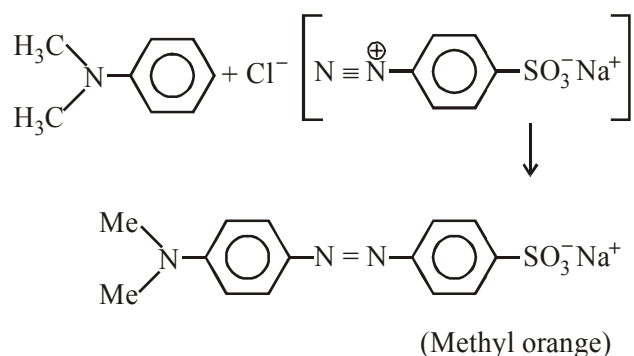


The product 'X' is used :

- (1) in acid base titration as an indicator
- (2) in protein estimation as an alternative to ninhydrin
- (3) in laboratory test for phenols
- (4) as food grade colourant

**NTA Ans. (1)**

**Sol.**



It is an acid base indicator

**19.** The IUPAC name of the complex

$[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NH}_2\text{CH}_3)]\text{Cl}$  is :

- (1) Diammine (methanamine) chlorido platinum (II) chloride
- (2) Bisammine (methanamine) chlorido platinum (II) chloride
- (3) Diamminechlorido (aminomethane) platinum(II) chloride
- (4) Diamminechlorido (methanamine) platinum (II) chloride

**NTA Ans. (4)**

**20.** Oxidation number of potassium in  $\text{K}_2\text{O}$ ,  $\text{K}_2\text{O}_2$  and  $\text{KO}_2$ , respectively, is :

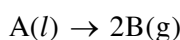
- (1) +1, +4 and +2
- (2) +1, +2 and +4
- (3) +1, +1 and +1

- (4) +2, +1 and  $+\frac{1}{2}$

**NTA Ans. (3)**

**Sol.** Potassium has an oxidation of +1 (only) in combined state.

**21.** For the reaction ;



$$\Delta U = 2.1 \text{ kcal}, \Delta S = 20 \text{ cal K}^{-1} \text{ at } 300 \text{ K}$$

Hence  $\Delta G$  in kcal is \_\_\_\_\_ .

**NTA Ans. (-2.70)**

**Sol.**  $\text{A}(\ell) \rightarrow 2\text{B}(\text{g})$

$$\Delta U = 2.1 \text{ Kcal}, \Delta S = 20 \text{ cal K}^{-1} \text{ at } 300 \text{ K}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = \Delta U + \Delta n_g RT - T\Delta S$$

$$= 2.1 + \frac{2 \times 2 \times 300}{1000} - \frac{300 \times 20}{1000}$$

$$(R = 2 \text{ cal K}^{-1} \text{ mol}^{-1})$$

$$= 2.1 + 1.2 - 6 = -2.70 \text{ Kcal/mol}$$

22. During the nuclear explosion, one of the products is  $^{90}\text{Sr}$  with half life of 6.93 years. if  $1\ \mu\text{g}$  of  $^{90}\text{Sr}$  was absorbed in the bones of a newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically\_\_\_\_\_ .

NTA Ans. (23 to 23.03)

Sol. All nuclear decays follow first order kinetics

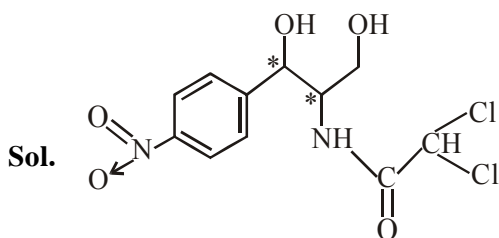
$$t = \frac{1}{k} \ln \frac{[A_0]}{[A]}$$

$$= \frac{(t_{1/2})}{0.693} \times 2.303 \log_{10} 10 = 10 \times 2.303 \times 1$$

$$= 23.03 \text{ years}$$

23. The number of chiral carbons in chloramphenicol is \_\_\_\_\_ .

NTA Ans. (2)



Chloramphenicol

24. Two solutions A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of  $\text{H}_2\text{SO}_4$  in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is\_\_\_\_\_.

NTA Ans. (10.60 to 10.60)

Sol. 4 gm of NaOH in 100 L sol.  $\Rightarrow 10^{-3}$  M sol.  
 9.8 gm of  $\text{H}_2\text{SO}_4$  in 100 L sol.  $\Rightarrow 10^{-3}$  M sol.  
 Mixture : 40L of  $10^{-3}$  M NaOH and 10 L of  $10^{-3}$  M  $\text{H}_2\text{SO}_4$  sol.  
 Final Conc. of  $\text{OH}^-$

$$= \frac{10^{-3}(40 \times 1 - 10 \times 1 \times 2)}{40 + 10} = 6 \times 10^{-4} \text{ M}$$

$$\text{pOH} = -\log(6 \times 10^{-4})$$

$$= 4 - \log 6 = 4 - 0.60 = 3.40$$

$$\text{pH} = 14 - 3.40 = 10.60$$

25. Chlorine reacts with hot and concentrated NaOH and produces compounds (X) and (Y). Compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is \_\_\_\_\_.

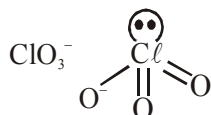
NTA Ans. (1.66 to 1.67)

Sol.  $3\text{Cl}_2 + 6\text{NaOH} \rightarrow 5\text{NaCl} + \text{NaClO}_3 + 3\text{H}_2\text{O}$

(X) (X)

$\text{NaCl} + \text{AgNO}_3 \rightarrow \text{AgCl} \downarrow + \text{NaNO}_3$

(X)



$$\text{Bond order of Cl-O Bond} = 1 + \frac{2}{3} = \frac{5}{3}$$

$$= 1.66 \text{ or } 1.67$$

**FINAL JEE-MAIN EXAMINATION – JANUARY, 2020**

(Held On Tuesday 07<sup>th</sup> JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

**MATHEMATICS**

**TEST PAPER WITH ANSWER & SOLUTION**

1. If  $g(x) = x^2 + x - 1$  and  $(g \circ f)(x) = 4x^2 - 10x + 5$ , then  $f\left(\frac{5}{4}\right)$  is equal to  
 (1)  $\frac{3}{2}$       (2)  $-\frac{1}{2}$       (3)  $-\frac{3}{2}$       (4)  $\frac{1}{2}$

NTA Ans. (2)

**Sol.**  $g(x) = x^2 + x - 1$   
 $g(f(x)) = 4x^2 - 10x + 5$   
 $= (2x - 2)^2 + (2 - 2x) - 1$   
 $= (2 - 2x)^2 + (2 - 2x) - 1$   
 $\Rightarrow f(x) = 2 - 2x$   
 $f\left(\frac{5}{4}\right) = \frac{-1}{2}$

2. If  $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$ , where  $z = x + iy$ , then the point  $(x, y)$  lies on a :

- (1) circle whose centre is at  $\left(-\frac{1}{2}, -\frac{3}{2}\right)$   
 (2) circle whose diameter is  $\frac{\sqrt{5}}{2}$   
 (3) straight line whose slope is  $\frac{3}{2}$   
 (4) straight line whose slope is  $-\frac{2}{3}$

NTA Ans. (2)

**Sol.**  $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$   
 Put  $z = x + iy$

$$\operatorname{Re}\left(\frac{(x+iy)-1}{2(x+iy)+i}\right) = 1$$

$$\operatorname{Re}\left(\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)\left(\frac{2x-i(2y+1)}{2x-i(2y+1)}\right)\right) = 1$$

$$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$$

$$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$$

$\Rightarrow$  locus is a circle whose

Centre is  $\left(-\frac{1}{2}, -\frac{3}{4}\right)$  and radius  $\frac{\sqrt{5}}{4}$

$$\Rightarrow \text{diameter} = \frac{\sqrt{5}}{2}$$

3. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is  $-\frac{1}{2}$ , then the greatest number amongst them is :

- (1)  $\frac{21}{2}$       (2) 27      (3) 16      (4) 7

NTA Ans. (3)

**Sol.** Let the A.P is

$$a - 2d, a - d, a, a + d, a + 2d$$

$$\therefore \text{sum} = 25 \Rightarrow a = 5$$

$$\text{Product} = 2520$$

$$(25 - 4d^2)(25 - d^2) = 504$$

$$4d^4 - 125d^2 + 121 = 0$$

$$\Rightarrow d^2 = 1, \frac{121}{4}$$

$$\Rightarrow d = \pm 1, \pm \frac{11}{2}$$

$d = \pm 1$  is rejected because none of the term can

be  $\frac{-1}{2}$ .

$$\Rightarrow d = \pm \frac{11}{2}$$

$$\Rightarrow \text{AP will be } -6, -\frac{1}{2}, 5, \frac{21}{2}, 16$$

Largest term is 16.

4. If

$$y(\alpha) = \sqrt{2 \left( \frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha} \right) + \frac{1}{\sin^2 \alpha}}, \alpha \in \left( \frac{3\pi}{4}, \pi \right),$$

then  $\frac{dy}{d\alpha}$  at  $\alpha = \frac{5\pi}{6}$  is :

- (1) 4      (2)  $-\frac{1}{4}$       (3)  $\frac{4}{3}$       (4) -4

NTA Ans. (1)

Sol.  $y(\alpha) = \sqrt{2 \left( \frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha} \right) + \frac{1}{\sin^2 \alpha}}, \alpha \in \left( \frac{3\pi}{4}, \pi \right)$

$$= \frac{|\sin \alpha + \cos \alpha|}{|\sin \alpha|} = \frac{-(\sin \alpha + \cos \alpha)}{\sin \alpha}$$

$$= -1 - \cot \alpha$$

$$y'(\alpha) = \operatorname{cosec}^2 \alpha$$

$$y' \left( \frac{5\pi}{6} \right) = 4$$

5. Let  $\alpha$  be a root of the equation  $x^2 + x + 1 = 0$

and the matrix  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$ , then the

matrix  $A^{31}$  is equal to:

- (1)  $A^3$       (2)  $A$       (3)  $A^2$       (4)  $I_3$

NTA Ans. (1)

Sol.  $x^2 + x + 1 = 0$

$$\alpha = \omega$$

$$\alpha^2 = \omega^2$$

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^4 = A^2 \cdot A^2 = I_3$$

$$A^{31} = A^{28} \cdot A^3 = A^3.$$

6. If  $y = mx + 4$  is a tangent to both the parabolas,  $y^2 = 4x$  and  $x^2 = 2by$ , then  $b$  is equal to :

- (1) 128      (2) -64      (3) -128      (4) -32

NTA Ans. (3)

Sol.  $y = mx + 4$  is tangent to  $y^2 = 4x$

$$\Rightarrow m = \frac{1}{4}$$

$$y = \frac{1}{4}x + 4 \text{ is tangent to } x^2 = 2by$$

$$\Rightarrow x^2 - \frac{b}{2}x - 8b = 0$$

$$\Rightarrow D = 0$$

$$b^2 + 128b = 0$$

$$\Rightarrow b = -128, 0$$

$$b \neq 0 \Rightarrow b = -128$$

7. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

- (1)  $\sqrt{3}$       (2)  $2\sqrt{3}$       (3)  $3\sqrt{2}$       (4)  $\frac{3}{\sqrt{2}}$

NTA Ans. (3)

Sol. Given  $2ae = 6 \Rightarrow \boxed{ae = 3} \dots\dots(1)$

and  $\frac{2a}{e} = 12 \Rightarrow \boxed{a = 6e} \dots\dots(2)$

from (1) and (2)

$$6e^2 = 3 \Rightarrow \boxed{e = \frac{1}{\sqrt{2}}}$$

$$\Rightarrow \boxed{a = 3\sqrt{2}}$$

Now,  $b^2 = a^2 (1 - e^2)$

$$\Rightarrow b^2 = 18 \left( 1 - \frac{1}{2} \right) = 9$$

$$\text{Length of L.R} = \frac{2(9)}{3\sqrt{2}} = 3\sqrt{2}$$



8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5 otherwise X takes the value -1. Then the expected value of X, is :

- (1)  $\frac{3}{16}$       (2)  $-\frac{3}{16}$       (3)  $\frac{1}{8}$       (4)  $-\frac{1}{8}$

**NTA Ans. (3)**

**Sol.**

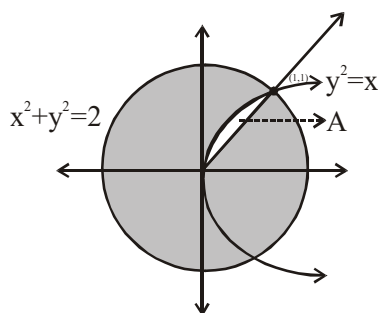
k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Expected value =  $\sum XP(k)$   
 $-\frac{1}{32} - \frac{12}{32} - \frac{11}{32} + \frac{15}{32} + \frac{8}{32} + \frac{5}{32}$   
 $= \frac{28-24}{32} = \frac{4}{32} = \frac{1}{8}$

9. The area of the region, enclosed by the circle  $x^2 + y^2 = 2$  which is not common to the region bounded by the parabola  $y^2 = x$  and the straight line  $y = x$ , is :

- (1)  $\frac{1}{3}(12\pi - 1)$       (2)  $\frac{1}{6}(12\pi - 1)$   
 (3)  $\frac{1}{6}(24\pi - 1)$       (4)  $\frac{1}{3}(6\pi - 1)$

**NTA Ans. (2)**



**Sol.**

$A = \int_0^1 (\sqrt{x} - x) dx$   
 $= \left[ \frac{2}{3} x^{3/2} - \frac{x^2}{2} \right]_0^1 = \frac{1}{6}$

Required Area :  $\pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$

10. Let  $x^k + y^k = a^k$ , ( $a, k > 0$ ) and  $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$ , then k is :

- (1)  $\frac{3}{2}$       (2)  $\frac{1}{3}$       (3)  $\frac{2}{3}$       (4)  $\frac{4}{3}$

**NTA Ans. (3)**

**Sol.**  $x^k + y^k = a^k$  ( $a, k > 0$ )

$kx^{k-1} + ky^{k-1} \frac{dy}{dx} = 0$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0 \Rightarrow k-1 = -\frac{1}{3} \Rightarrow k = 2/3$

11. If  $y = y(x)$  is the solution of the differential

equation,  $e^y \left(\frac{dy}{dx} - 1\right) = e^x$  such that  $y(0) = 0$ , then

$y(1)$  is equal to :

- (1)  $2 + \log_e 2$       (2)  $2e$   
 (3)  $\log_e 2$       (4)  $1 + \log_e 2$

**NTA Ans. (4)**

**Sol.**  $e^y \frac{dy}{dx} - e^y = e^x$ , Let  $e^y = t$

$\Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

I.F. =  $e^{\int -dx} = e^{-x}$

$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

12. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is :

- (1)  $\frac{5}{2}(6!)$       (2)  $5^6$       (3)  $\frac{1}{2}(6!)$       (4)  $6!$

**NTA Ans. (1)**

**Sol.** Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

$${}^5C_1 \times \frac{6!}{2!}$$

**13.** Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

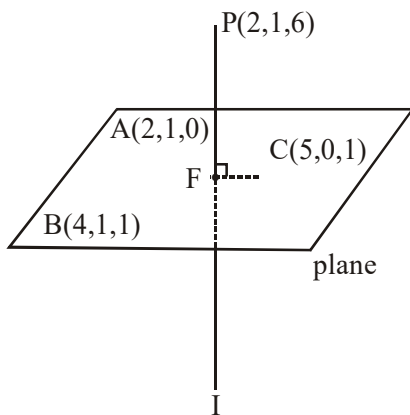
- (1) (6, 5, -2)                      (2) (4, 3, 2)  
 (3) (3, 4, -2)                      (4) (6, 5, 2)

**NTA Ans. (1)**

**Sol.** Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x + y - 2z = 3$$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

$$\text{eqn of line PI } \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda (\text{say})$$

Let I  $(\lambda + 2, \lambda + 1, -2\lambda + 6)$

$$\Rightarrow F \left( 2 + \frac{\lambda}{2}, 1 + \frac{\lambda}{2}, -\lambda + 6 \right)$$

F lies in the plane

$$\Rightarrow 2 + \frac{\lambda}{2} + 1 + \frac{\lambda}{2} + 2\lambda - 12 - 3 = 0$$

$$\Rightarrow \lambda = 4$$

$$\Rightarrow I (6, 5, -2)$$

**14.** A vector  $\vec{a} = \alpha \hat{i} + 2\hat{j} + \beta \hat{k}$  ( $\alpha, \beta \in \mathbb{R}$ ) lies in the plane of the vectors  $\vec{b} = \hat{i} + \hat{j}$  and  $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ .

If  $\vec{a}$  bisects the angle between  $\vec{b}$  and  $\vec{c}$ , then:

(1)  $\vec{a} \cdot \hat{i} + 1 = 0$                       (2)  $\vec{a} \cdot \hat{i} + 3 = 0$

(3)  $\vec{a} \cdot \hat{k} + 4 = 0$                       (4)  $\vec{a} \cdot \hat{k} + 2 = 0$

**NTA Ans. (4)**

**Sol.**  $\vec{a} = \lambda(\vec{b} + \vec{c}) = \lambda \left( \frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}} \right)$

$$\vec{a} = \frac{\lambda}{3\sqrt{2}} (4\hat{i} + 2\hat{j} + 4\hat{k}) \Rightarrow \frac{\lambda}{3\sqrt{2}} (4\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= \alpha \hat{i} + 2\hat{j} + \beta \hat{k}$$

$$\Rightarrow \alpha = 4 \text{ and } \beta = 4$$

$$\text{So, } \vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

None of the given options is correct

**15.** If  $f(a + b + 1 - x) = f(x)$ , for all x, where a and b are fixed positive real numbers, then

$$\frac{1}{a+b} \int_a^b x(f(x) + f(x+1)) dx \text{ is equal to :}$$

(1)  $\int_{a+1}^{b+1} f(x) dx$                       (2)  $\int_{a+1}^{b+1} f(x+1) dx$

(3)  $\int_{a-1}^{b-1} f(x+1) dx$                       (4)  $\int_{a-1}^{b-1} f(x) dx$

**NTA Ans. (1)**



**Sol.** For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$$\Rightarrow (3b - 2a)(c - a) - (b - a)(4c - 2a) = 0$$

$$\Rightarrow 2ac = bc + ab$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Hence  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in A.P.

**18.** Let  $\alpha$  and  $\beta$  be two real roots of the equation  $(k + 1) \tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$ , where  $k (\neq -1)$  and  $\lambda$  are real numbers. If  $\tan^2(\alpha + \beta) = 50$ , then a value of  $\lambda$  is ;

- (1) 5      (2) 10      (3)  $5\sqrt{2}$       (4)  $10\sqrt{2}$

**NTA Ans. (2)**

**Sol.**  $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$$

$$\tan(\alpha + \beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \text{ \& } -10$$

**19.** The logical statement  $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$  is equivalent to :

- (1) p      (2) q      (3)  $\sim p$       (4)  $\sim q$

**NTA Ans. (3)**

**Sol.**  $(p \rightarrow q) \wedge (q \rightarrow \sim p)$   
 $\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$   
 $\equiv \sim p \vee (q \wedge \sim q)$   
 $\equiv \sim p \vee C \equiv \sim p$

**20.** The greatest positive integer k, for which  $49^k + 1$  is a factor of the sum

$$49^{125} + 49^{124} + \dots + 49^2 + 49 + 1, \text{ is :}$$

- (1) 32      (2) 60      (3) 63      (4) 65

**NTA Ans. (3)**

**Sol.**  $1 + 49 + 49^2 + \dots + 49^{12}$

$$= (49)^{126} - 1 = (49^{63} + 1) \frac{(49^{63} - 1)}{(48)}$$

So greatest value of k = 63

**21.**  $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$  is equal to \_\_\_\_\_.

**NTA Ans. (36)**

**Sol.**  $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} \Rightarrow \lim_{x \rightarrow 2} \frac{3^{2x} - 12 \cdot 3^x + 27}{3^{x/2} - 3}$

$$= \lim_{x \rightarrow 2} \frac{(3^x - 9)(3^x - 3)}{(3^{x/2} - 3)}$$

$$= \lim_{x \rightarrow 2} \frac{(3^{x/2} + 3)(3^{x/2} - 3)(3^x - 3)}{(3^{x/2} - 3)}$$

$$= 36$$

**22.** If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to \_\_\_\_\_.

**NTA Ans. (18)**

**Sol.** Variance of first 'n' natural numbers =  $\frac{n^2-1}{12} = 10$

$\Rightarrow n = 11$

and variance of first 'm' even natural numbers

$= 4\left(\frac{m^2-1}{12}\right) \Rightarrow \frac{m^2-1}{3} = 16 \Rightarrow m = 7$

$m + n = 18$

**23.** If the sum of the coefficients of all even powers of x in the product

$(1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 - x^3 + \dots + x^{2n})$  is 61, then n is equal to \_\_\_\_\_.

**NTA Ans. (30)**

**Sol.** Let  $(1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 - x^3 + \dots + x^{2n})$   
 $= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \dots + a_{4n} = 2n + 1 \quad \dots(1)$

$a_0 - a_1 + a_2 - a_3 + \dots + a_{4n} = 2n + 1 \quad \dots(2)$

$\Rightarrow a_0 + a_2 + a_4 + \dots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61 \quad \Rightarrow n = 30$

**24.** Let A(1, 0), B(6, 2) and C( $\frac{3}{2}, 6$ ) be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the

line segment PQ, where Q is the point ( $-\frac{7}{6}, -\frac{1}{3}$ ), is \_\_\_\_\_.

**NTA Ans. (5)**

**Sol.** P is centroid of the triangle ABC

$\Rightarrow P \equiv \left(\frac{17}{6}, \frac{8}{3}\right)$

$\Rightarrow PQ = 5$

**25.** Let S be the set of points where the function,  $f(x) = |2 - |x - 3||$ ,  $x \in \mathbb{R}$ , is not differentiable.

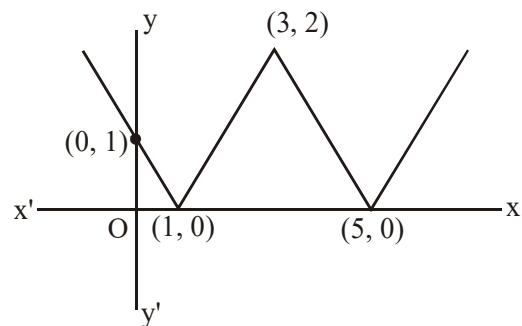
Then  $\sum_{x \in S} f(f(x))$  is equal to \_\_\_\_\_.

**NTA Ans. (3)**

**Sol.**  $f(x) = |2 - |x - 3||$

f is not differentiable at

$x = 1, 3, 5$



$\Rightarrow \sum_{x \in S} f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1 = 3$