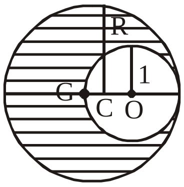
# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM**

## PHYSICS

**TEST PAPER WITH ANSWER & SOLUTION**

**1.** As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e, on the surface of the cavity. R can be detemined by the equation :




(1) $(R^2 - R + 1)(2 - R) = 1$

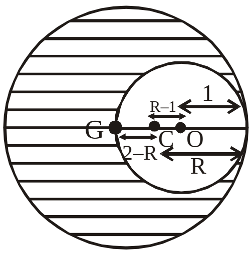
(2) $(R^2 + R - 1)(2 - R) = 1$

(3) $(R^2 + R + 1)(2 - R) = 1$

(4) $(R^2 - R - 1)(2 - R) = 1$

**NTA Ans. (3)**

**Sol.**




By concept of COM

$m_1R_1 = m_2R_2$

Remaining mass × (2–R) = cavity mass × (R–1)

$$\left(\frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi 1^3\rho\right)(2-R) = \frac{4}{3}\pi 1^3\rho \times (R-1)$$

$(R^3 - 1)(2 - R) = R - 1$

$(R^2 + R + 1)(2 - R) = 1$

**2.** In a double slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is :

(1) 0.568 (2) 0.672 (3) 0.760 (4) 0.853

**NTA Ans. (4)**

**Sol.** $I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$$\frac{I}{I_0} = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\frac{2\pi}{\lambda}\times\frac{\lambda}{8}}{2}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = 0.853$$

**3.** A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5\times10^{-8}\hat{j}\,T$. The corresponding electric field $\vec{E}$ is (speed of light $c = 3 \times 10^8$ ms$^{-1}$)

(1) $1.66 \times 10^{-16}\ \hat{i}\ V/m$

(2) $15\ \hat{i}\ V/m$

(3) $-1.66\times10^{-16}\ \hat{i}\ V/m$

(4) $-15\ \hat{i}\ V/m$

**NTA Ans. (2)**

**Sol.** $E = \vec{B} \times \vec{V}$

$= (5\times10^{-8}\hat{j}) \times (3\times10^{8}\hat{k})$

$= 15\ \hat{i}\ V/m$

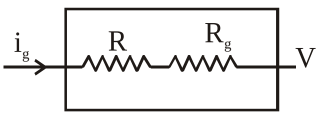
**4.** A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?

(1) 9.9 kΩ (2) 8.9 kΩ

(3) 7.9 kΩ (4) 10 kΩ

**NTA Ans. (1)**

**Sol.** $i_g = 1\ \text{mA}$ , $R_g = 100\ \Omega$

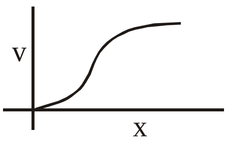


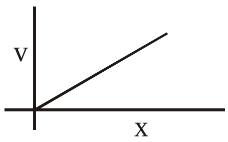

$V = i_g(R + R_g)$

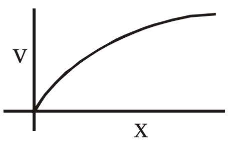
$10 = 1 \times 10^{-3}\ (R + 100)$

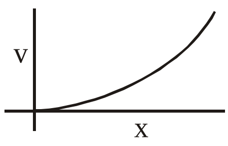
$R = 9.9\ \text{k}\Omega$

**5.** A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

(1) 


(2) 


(3) 

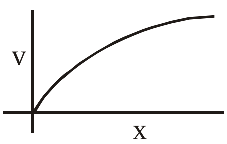

(4) 


**NTA Ans. (3)**

**Sol.** $v^2 = u^2 + 2as$

$v^2 = 0 + 2\left(\frac{qE}{m}\right)x$

$v^2 = \frac{2qE}{m}x$




**6.** A simple pendulum is being used to determine th value of gravitational acceleration g at a certain place. Th length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is :

(1) 3.40% (2) 5.40%

(3) 4.40% (4) 2.40%

**NTA Ans. (2)**

**Sol.** $T = 2\pi\sqrt{\frac{\ell}{g}}$

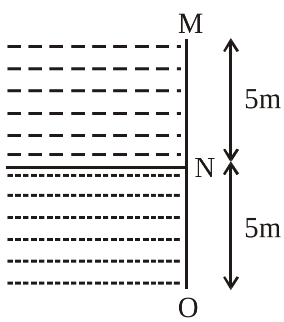
$$g = \frac{4\pi^2\ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \frac{0.1}{25} + \frac{2 \times 1}{50}$$

$$\frac{\Delta g}{g} = 4.4\%$$

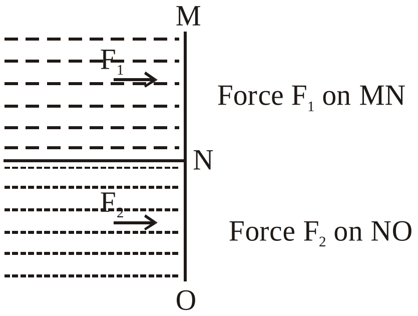
**7.** Two liquids of densities $\rho_1$ an $\rho_2$ ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)




(1) 1/4 (2) 2/3 (3) 1/3 (4) 1/2

**NTA Ans. (1)**

**Sol.**




$$F_1 = \frac{\rho g h}{2} \times A$$

$$F_2 = \left(\rho g h + \frac{2\rho g h}{2}\right) A$$

$$\frac{F_1}{F_2} = \frac{1}{4}$$

**8.** A transverse wave travels on a taut steel wire with a velocity of v when tension in it is $2.06 \times 10^4$ N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to :

(1) $10.2 \times 10^2$ N (2) $5.15 \times 10^3$ N

(3) $2.50 \times 10^4$ N (4) $30.5 \times 10^4$ N

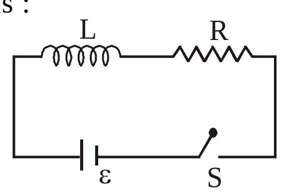
**NTA Ans. (2)**

**Sol.** Velocity of transverse wave $V \propto \sqrt{T}$

$$V \to \frac{V}{2} \Rightarrow T \to T' = \frac{T}{4}$$

$$T' = \frac{2.06 \times 10^4}{4} = 5.15 \times 10^3 N$$

**9.** A shown in the figure, a battery of emf $\varepsilon$ is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and t = $t_c$ ($t_c$ is the time constant of the circuit) is :




(1) $\frac{\varepsilon L}{R^2}\left(1 - \frac{1}{e}\right)$ (2) $\frac{\varepsilon R}{eL^2}$

(3) $\frac{\varepsilon L}{R^2}$ (4) $\frac{\varepsilon L}{eR^2}$

**NTA Ans. (4)**

**Sol.** $i = i_0 (1 - e^{-Rt/L}) = i_0 (1 - e^{-t/T_C})$

$$q = \int_0^{T_C} i \, dt$$

$$= \int_0^{T_C} \frac{\varepsilon}{R}\left(1 - e^{-t/T_C}\right)$$

$$= \frac{\varepsilon}{R}\left(t - \frac{e^{-t/T_C}}{-1/T_C}\right)\Bigg|_0^{T_C}$$

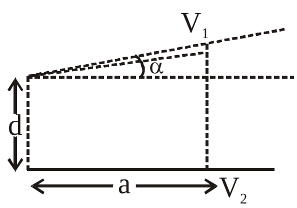
$$= \frac{e}{R}\left(T_C - T_C e^{-1}\right) - \frac{e}{R}(0 + T_C)$$

$$q = \frac{e}{R} \times T_C \; e^{-1}$$

$$= \frac{\varepsilon}{R} \times \frac{L}{R} \frac{1}{e}$$

$$= \frac{\varepsilon L}{eR^2}$$

**10.** A capacitor is made of two square plates each of side 'a' making a very small angle $\alpha$ between them, as shown in figure. The capacitance will be close to :

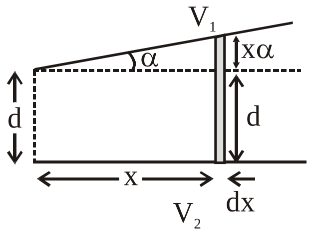



(1) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{3\alpha a}{2d}\right)$ (2) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{4d}\right)$

(3) $\frac{\varepsilon_0 a^2}{d}\left(1 + \frac{\alpha a}{d}\right)$ (4) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$

**NTA Ans. (4)**

**Sol.** Assume small element dx at a distance x from left end




Capacitance for small element dx is

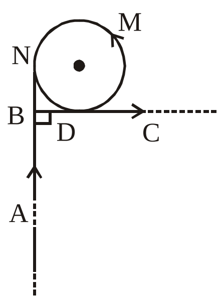
$$dC = \frac{\varepsilon_0 a\,dx}{d + x\alpha}$$

$$C = \int_0^a \frac{\varepsilon_0 a\,dx}{d + x\alpha}$$

$$= \frac{\varepsilon_0 a}{\alpha}\ln\left(\frac{1+\alpha a}{d}\right)\Bigg|_0^a \qquad \left(\ln(1+x) \approx x - \frac{x^2}{2}\right)$$

$$= \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

**11.** A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is :

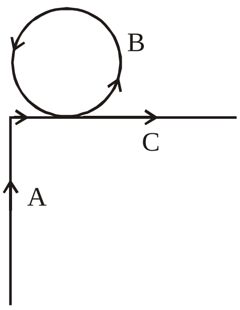



(1) $\frac{\mu_0 I}{2R}$ (2) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$

(3) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$ (4) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$

**NTA Ans. (3)**

**Sol.** We say we have 3 parts (A, B, C)




$B = B_A + B_B + B_C$

$$= \frac{\mu_0 I}{4\pi R}(\sin 90° - \sin 45°) \otimes + \frac{\mu_0 I}{2R} \odot + \frac{\mu_0 I}{4\pi R}(\sin 45° + \sin 90°) \odot$$

$$= \frac{\mu_0 I}{2\pi R}(\sin 45° + \pi)$$

$$= \frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$$

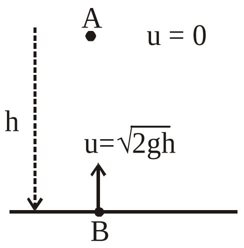
**12.** A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{1}{2}}$

(3) $\sqrt{\frac{3}{4}}$ (4) $\sqrt{\frac{3}{2}}$

**NTA Ans. (4)**

**Sol.**

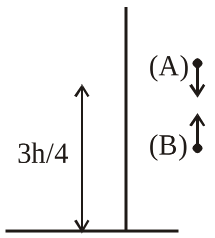


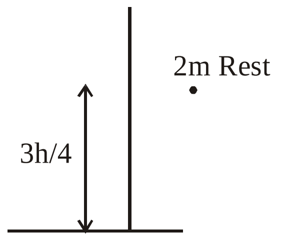

Particles will collide after time $t_0 = \dfrac{h}{\sqrt{2gh}}$

at collision, $v_A = gt_0$ $\qquad v_B = u_B - gt_0$

$\Rightarrow v_A = -v_B$

Before collision $\qquad$ After collision




Time taken by combined mass to reach the ground

$$\text{time} = \sqrt{\frac{2\times 3h/4}{g}} = \sqrt{\frac{3h}{2g}}$$

**13.** A carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is :

(1) 99 J $\qquad$ (2) 100 J

(3) 90 J $\qquad$ (4) 1 J

**NTA Ans. (3)**

**Sol.** Refrigerator cycle is :

$$\eta = \frac{W}{Q_+} = \frac{W}{W+Q_-}$$

$$\frac{1}{10} = \frac{10}{10+Q_-}$$

$Q_- = 90$ J

Heat absorbed from the reservoir at lower temperature is 90 J

**14.** Consider a mixture of n moles of helium gas and 2 n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its $C_P/C_V$ value will be :

(1) 67/45 $\qquad$ (2) 19/13

(3) 23/15 $\qquad$ (4) 40/27

**NTA Ans. (2)**

**Sol.** $$\frac{C_P}{C_V}\text{mix} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}}$$

$$\frac{C_P}{C_V}\text{mix} = \frac{n\times\left(\frac{5R}{2}\right) + 2n\left(\frac{7R}{2}\right)}{n\times\frac{3R}{2} + 2n\left(\frac{5R}{2}\right)}$$

$$\frac{C_P}{C_V} = \frac{19}{13}$$

**15.** An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If $\lambda_0$ is initial de-Broglie wavelength of electron, its de-Broglie wave length at time t is given by :

(1) $\dfrac{\lambda_0\sqrt{2}}{\sqrt{1+\dfrac{e^2E^2t^2}{m^2v_0^2}}}$ $\qquad$ (2) $\dfrac{\lambda_0}{\sqrt{2+\dfrac{e^2E^2t^2}{m^2v_0^2}}}$

(3) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E^2t^2}{2m^2v_0^2}}}$ $\qquad$ (4) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$

**NTA Ans. (3)**

**Sol.** By de-Broglie hypothesis

$$\lambda = \frac{h}{mv}$$

$$\lambda_0 = \frac{h}{m\sqrt{2}v_0} \quad \text{........(1)}$$

$$\lambda' = \frac{h}{\sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0t}{m}\right)^2}}$$

$$= \frac{h}{m\sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}} \quad \text{........(2)}$$

By (1) and (2)

$$\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

**16.** A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is :

(1) $8.75 \times 10^{-4}$ J  (2) $8.75 \times 10^{-3}$ J

(3) $6.25 \times 10^{-4}$ J  (4) $1.13 \times 10^{-3}$ J

**NTA Ans. (1)**

**Sol.** m = 0.5 kg, v = 5 cm/s

KE in rolling = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$= 8.75 \times 10^{-4}$ J

**17.** Consider two charged metallic spheres $S_1$ and $S_2$ of radii $R_1$ and $R_2$, respectively. The electric fields $E_1$ (on $S_1$) and $E_2$ (on $S_2$) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio $V_1$ (on $S_1$) / $V_2$ (on $S_2$) of the electrostatic potentials on each sphere is :

(1) $(R_2/R_1)$  (2) $\left(\frac{R_1}{R_2}\right)^3$

(3) $R_1/R_2$  (4) $(R_1/R_2)^2$

**NTA Ans. (4)**

**Sol.** $E_1 = \frac{KQ_1}{R_1^2}$  $E_2 = \frac{KQ_2}{R_2^2}$

Given,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

$$\frac{\frac{KQ_1}{R_1^2}}{\frac{KQ_2}{R_2^2}} = \frac{R_1}{R_2} \Rightarrow \boxed{\frac{Q_1}{Q_2} = \frac{R_1^3}{R_2^3}}$$

$$\frac{V_1}{V_2} = \frac{KQ_1/R_1}{KQ_2/R_2} = \frac{R_1^2}{R_2^2}$$

**18.** A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where $\omega$ is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle :

(1) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

(2) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(3) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

**NTA Ans. (1)**

**Sol.** $\vec{r}(t) = \cos\omega \hat{i} + \sin\omega t\,\hat{j}$

On diff. we get

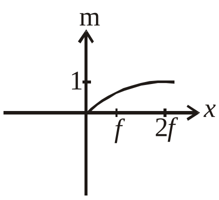
$\vec{v} = -\omega\sin\omega t\ \hat{i} + \omega\cos\omega t\ \hat{j}$

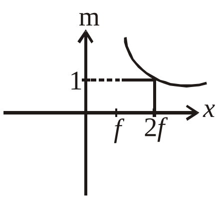
$\vec{a} = -\omega^2\vec{r}$

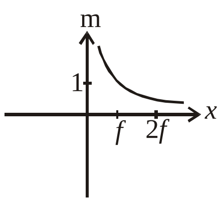
$\vec{v}\,.\,\vec{r} = 0$

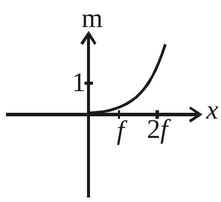
**19.** An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by :

(Graphs are drawn schematically and are not to scale)

(1) 


(2) 


(3) 


(4) 

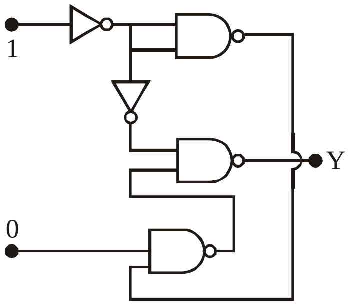

**NTA Ans. (2)**

**Sol.** $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

At focus m = ∞ x = f

At centre m = –1 x = 2f

**20.** In the given circuit, value of Y is :




(1) will not execute

(2) 0

(3) toggles between 0 and 1

(4) 1

**NTA Ans. (2)**

**Sol.** Y $= \overline{\overline{AB}.A}$

$= \overline{\overline{AB}} + \overline{A}$

= 0 + 0

= 0

**21.** Three containers $C_1$, $C_2$ and $C_3$ have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in litres) are taken from each containers and mixed (assume no loss of heat during the process)

| $C_1$ | $C_2$ | $C_3$ | T |
|---|---|---|---|
| 1*l* | 2*l* | – | 60°C |
| – | 1*l* | 2*l* | 30°C |
| 2*l* | – | 1*l* | 60°C |
| 1*l* | 1*l* | 1*l* | θ |

The value of θ (in °C to the nearest integer) is ..........

**NTA Ans. (50)**

**Sol.** According to table and applying law of calorimetry

$1T_1 + 2T_2 = (1 + 2)60°$ ........(1)

$= 180$

$1T_2 + 2T_3 = (1 + 2)30°$ ........(2)

$= 90$

$2T_1 + 1T_3 = (1 + 2)60$

$= 180$ ........(3)

Adding (1) + (2) + (3)

$3 (T_1 + T_2 + T_3) = 450$

$T_1 + T_2 + T_3 = 150°$

Hence,

$T_1 + T_2 + T_3 = (1 + 1 + 1)\theta$

$150 = 3\theta$

$\theta = 50°C$

**22.** A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in $ms^{-2}$) near the surface on that planet is ______

**NTA Ans. (8 or 2888)**

**Sol.** Time to travel 81 m is t sec.

Time to travel 100 m is t + $\frac{1}{2}$ sec.

$81 = \frac{1}{2}\times a\times t^2 \qquad \Rightarrow t = 9\sqrt{\frac{2}{a}}$

$100 = \frac{1}{2}\times a\times\left(t+\frac{1}{2}\right)^2 \qquad \Rightarrow t + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$

$9\sqrt{\frac{2}{a}}+\frac{1}{2}=10\sqrt{\frac{2}{a}}$

$\frac{1}{2}=\sqrt{\frac{2}{a}}$

$\boxed{a=8\ m/s^2}$

**23.** The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is:

**NTA Ans. (486)**

**Sol.** For Balmer series,

$$\frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n_2^2}\right)$$

$$\frac{\lambda_2}{\lambda_1}=\frac{\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{\left(\frac{1}{2^2}-\frac{1}{4^2}\right)}$$

$$\frac{\lambda_2}{6561}=\frac{5/36}{3/16}$$

$$\lambda_2 = \frac{20}{27}\times 6561$$

$\lambda_2 = 4860$ Å

$= 486$ nm

**24.** An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s) ? Give your answer to the nearest integer in kilometer/s _____.

**NTA Ans. (16)**

**Sol.** $U_1 + K_1 = U_2 + K_2$

$$-\frac{GM_em}{10R}+\frac{1}{2}mv_0^2=-\frac{GM_em}{R}+\frac{1}{2}mv^2$$

$$+\frac{9}{10}\times\frac{GM_em}{R}+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\frac{9}{10}\times\frac{1}{2}M\times v_e^2+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$v^2 = \frac{9}{10}v_e^2+v_0^2$$

$$= \frac{9}{10}\times(11.2)^2+(12)^2$$

$v^2 = 112.896 + 144$

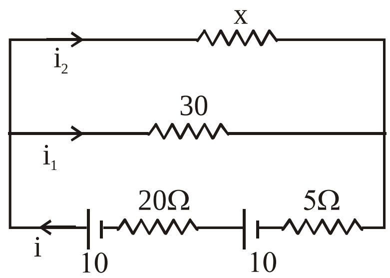
$v = 16.027$

$v = 16$ km/s

**25.** The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20Ω and 5Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is : _______

**NTA Ans. (30)**

**Sol.**




$E_1 = E - ir$

$= 10 - i20 = 0$

$i = 0.5$ A

$E_2 = E - ir$

$= 10 - 0.5 \times 5$

$= 7.5$ V

$$E_{net} = E_1 + E_2 = 7.5V$$

$i = i_1 + i_2$

$$0.5 = \frac{7.5}{x}+\frac{7.5}{30}$$

$x = 30\ \Omega$

# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM**

## CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

**1.** Among the reactions (a) - (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are :

(a) $CaO + SiO_2 \rightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

(c) $FeO + SiO_2 \rightarrow FeSiO_3$

(d) $FeO \rightarrow Fe + \frac{1}{2}O_2$

(1) (c) and (d) (2) (a) and (d)

(3) (d) (4) (a)

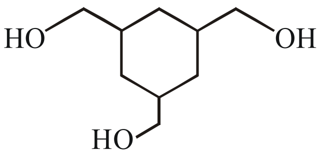
**NTA Ans. (1)**

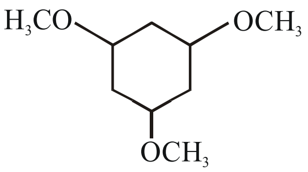
**Sol.** In blast furnace (metallugy of iron) involved reactions are

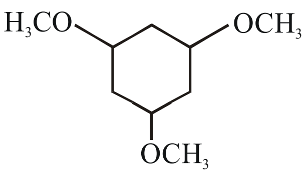
(a) $CaO + SiO_2 \longrightarrow CaSiO_3$

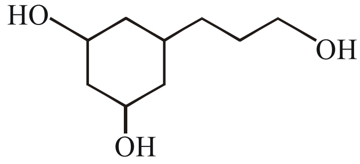
(b) $3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$

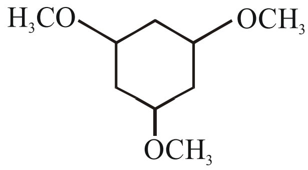
**2.** Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point the B. The possible structures of A and B are :

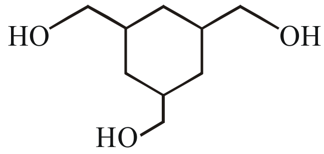
A = 

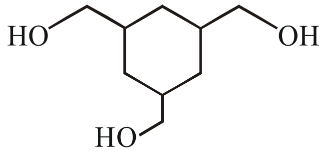
(1) B = 

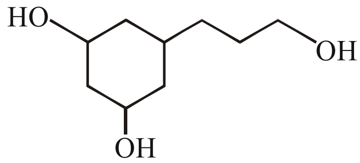
A = 

(2) B = 

A = 

(3) B = 

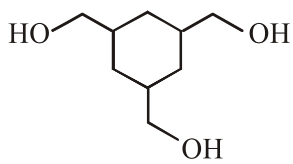
A = 

(4) B = 

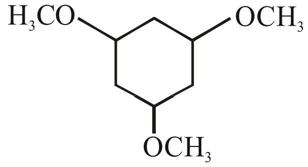
**NTA Ans. (1)**

**Sol.** Alcohol has more boiling point than ether (due to hydrogen bonding).

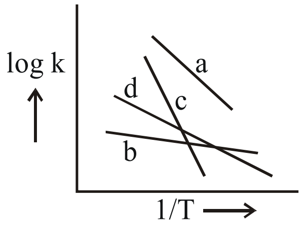
So,



has more boiling point than



**3.** Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?




(1) $E_b > E_d > E_c > E_a$ (2) $E_a > E_c > E_d > E_b$
(3) $E_c > E_a > E_d > E_b$ (4) $E_b > E_a > E_d > E_c$

**NTA Ans. (3)**

**Sol.** $\log K = \frac{-Ea}{2.303RT} + \log A$

Acrroding to Arrhenius equation plot of 'log K' Vs. $\frac{1}{T}$ is linear with.

Slope = $\frac{-Ea}{2.303R}$

From plot we conclude :

$\underset{(\text{magnitude})}{|\text{slope}|}$ : c > a > d > b

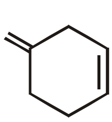
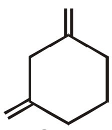
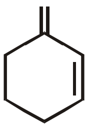
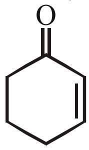
$\therefore\ E_c > E_a > E_d > E_b$

**4.** An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenattion, and also gives following reaction :

$$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$$

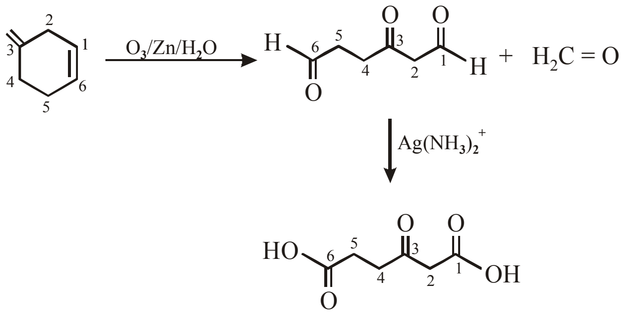
B(3 - oxo - hexanedicarboxylic acid)

X will be :-

(1) 
(2) 
(3) 
(4) 


**NTA Ans. (1)**

**Sol.**




**5.** The increasing order of the atomic radii of the following elements is :-

(a) C (b) O (c) F (d) Cl
(e) Br

(1) (b) < (c) < (d) < (a) < (e)
(2) (a) < (b) < (c) < (d) < (e)
(3) (d) < (c) < (b) < (a) < (e)
(4) (c) < (b) < (a) < (d) < (e)

**NTA Ans. (4)**

**Sol.** If the given elements are arranged according to their position in periodic table

Atomic radius

$$\begin{array}{c} C > O > F \\ \qquad\quad\ \wedge \\ \qquad\quad Cl \\ \qquad\quad\ \wedge \\ \qquad\quad Br \end{array}$$

$Br > Cl > C > O > F$

c < b < a < d < e

**6.** Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

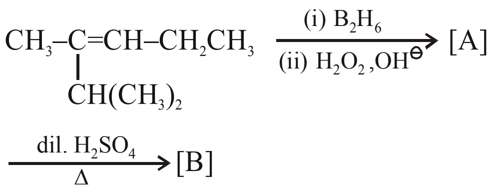
(1) $C_6H_5NO_2$ (2) $C_6H_5NH_2$

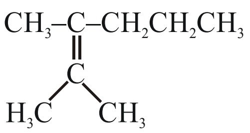
(3) $CH_3CH_2–C\equiv N$ (4) $NH_2–\overset{\overset{\displaystyle O}{\|}}{C}–NH_2$

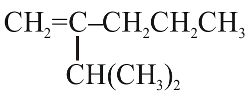
**NTA Ans. (1)**

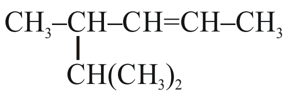
**Sol.** Kjeldahl's method for estimation of nitrogen is not applicable for nitrobenzene $C_6H_5NO_2$. because reaction with $H_2SO_4$, nitrobenzene can not give ammonia.

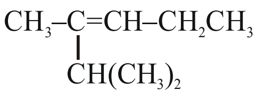
**7.** The major product [B] in the following sequence of reactions is :-



(1) 

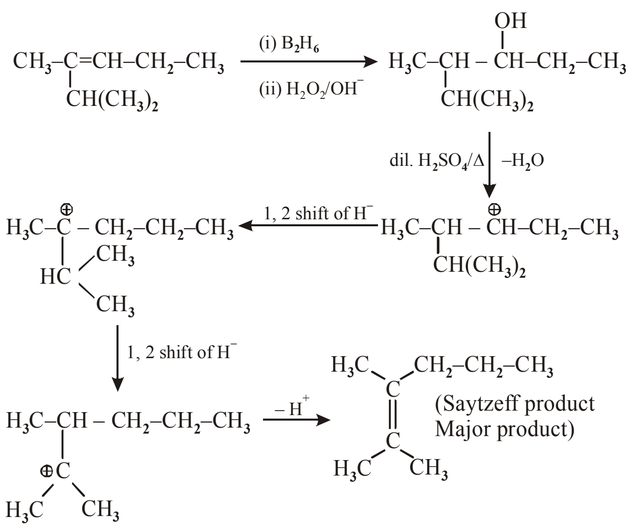
(2) 

(3) 

(4) 

**NTA Ans. (1)**

**Sol.**




**8.** A metal (A) on heating in nitrogen gas gives compound B. B on treatment with $H_2O$ gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are :

(1) Mg and $Mg_3N_2$

(2) Na and $NaNO_3$

(3) Mg and $Mg(NO_3)_2$

(4) Na and $Na_3N$

**NTA Ans. (1)**

**Sol.** $\underset{(A)}{3Mg} + N_2 \xrightarrow{\Delta} \underset{(B)}{Mg_3N_2}$

$Mg_3N_2 \xrightarrow{6H_2O} 3Mg(OH)_2 + \underset{\text{colourless gas}}{2NH_3}$

$CuSO_4 + 4NH_3 \longrightarrow \underset{\text{deep blue solution}}{[Cu(NH_3)_4]SO_4}$

**9.** Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form?

(1) AgBr (2) ZnS (3) KBr (4) CsCl

**NTA Ans. (1)**

**Sol.** Since AgBr has intermediate radius ratio

$\therefore$ it shows both schottky & Frenkel defects

ZnS $\rightarrow$ Frenkel defects

KBr, CsCl $\rightarrow$ Schottky defects

**10.** For the following Assertion and Reason, the correct option is :

Assertion : The pH of water increases with increase in temperature.

Reason : The dissociation of water into $H^+$ and $OH^-$ is an exothermic reaction.

(1) Both assertion and reason are true, but the reason is not the correct explanation for the assertion.

(2) Both assertion and reason are false.

(3) Assertion is not true, but reason is true.

(4) Both assertion and reason are true, and the reason is the correct explanation for the assertion.

**NTA Ans. (2)**

**Sol.** $H_2O(\ell) \rightleftharpoons H^+_{(aq)} + OH^-_{(aq)}$

For ionization of $H_2O$ : $\Delta H > O$

$\Rightarrow$ ENDOTHERMIC

On temperature increase reaction shifts forward

$\Rightarrow$ both $[H^+]$ and $[OH^-]$ increase

$\Rightarrow$ pH & pOH decreases.

**11.** Arrange the following bonds according to their average bond energies in descending order :

C–Cl, C–Br, C–F, C–I

(1) C–I > C–Br > C–Cl > C–F

(2) C–Br > C–I > C–Cl > C–F

(3) C–F > C–Cl > C–Br > C–I

(4) C–Cl > C–Br > C–I > C–F

**NTA Ans. (3)**

**Sol.** Bond length order in carbon halogen bonds are in the order of C – F < C – Cl < C – Br < C – I

Hence, Bond energy order

C – F > C – Cl > C – Br > C – I

**12.** White Phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of $CO_2$ gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is :

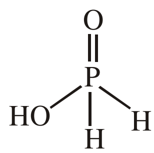
(1) 4 (2) 1

(3) 2 (4) 3

**NTA Ans. (2)**

**Sol.** $P_4 + 3NaOH + 3H_2O \longrightarrow \underset{(X)}{3NaH_2PO_2} + PH_3$

$NaH_2PO_2 + HCl \longrightarrow NaCl + \underset{(Y)}{H_3PO_2}$



(mono basic acid)

**13.** The radius of the second Bohr orbit, in terms of the Bohr radius, $a_0$, in $Li^{2+}$ is :

(1) $\frac{4a_0}{9}$ (2) $\frac{2a_0}{9}$

(3) $\frac{2a_0}{3}$ (4) $\frac{4a_0}{3}$

**NTA Ans. (4)**

**Sol.** $r_n = \frac{n^2 \times a_0}{z}$

For $2^{nd}$ Bohr orbit of $Li^{+2}$

n = 2

z = 3

$\Rightarrow r_n = \frac{2^2 \times a_0}{3} = \frac{4a_0}{3}$

**14.** Among (a) – (d) the complexes that can display geometrical isomerism are :

(a) $[Pt(NH_3)_3Cl]^+$ (b) $[Pt(NH_3)Cl_5]^-$

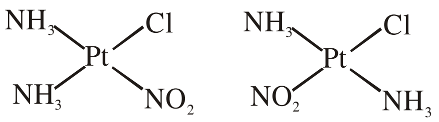
(c) $[Pt(NH_3)_2Cl(NO_2)]$ (d) $[Pt(NH_3)_4ClBr]^{2+}$

(1) (d) and (a) (2) (a) and (b)

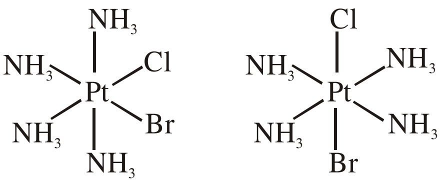
(3) (b) and (c) (4) (c) and (d)

**NTA Ans. (4)**

**Sol.** $[Pt(NH_3)_3Cl]^+$ & $[Pt(NH_3)Cl_5]^-$ does not show geometrical isomerism

(c) 

cis trans

(d) 

cis trans

**15.** Two monomers in maltose are :

(1) α-D-glucose and β-D-glucose

(2) α-D-glucose and α-D-Fructose

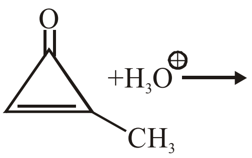
(3) α-D-glucose and α-D-glucose

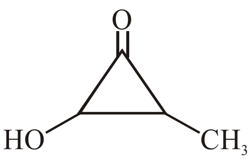
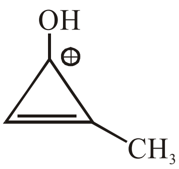
(4) α-D-glucose and α-D-galactose

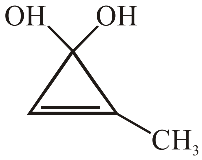
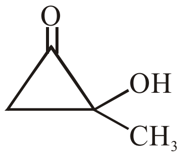
**NTA Ans. (3)**

**Sol.** Two monomers in maltose are α-D-glucose & α-D-glucose.

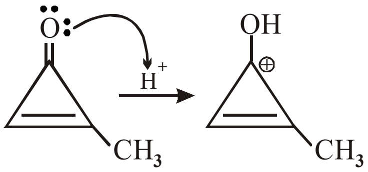
**16.** The major product in the following reaction is:



(1)  (2) 

(3)  (4) 

**NTA Ans. (2)**

**Sol.** 

(Aromatic stable product)

**17.** Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) an (z) is :

(1) 4 (2) 3 (3) 2 (4) 1

**NTA Ans. (2)**

**Sol.** Hydrogen has three isotopes

| Isotopes | Number of neutrons |
|---|---|
| Protium ($^1_1H$) | 0 |
| Deutrium ($^2_1H$) | 1 |
| Tritium ($^3_1H$) | 2 |

Hence the sum of neutrons are 3

**18.** Preparation of Bakelite proceeds via reactions.

(1) Condensation and elimination

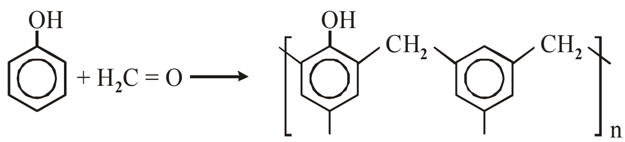
(2) Electrophilic addition and dehydration

(3) Electrophilic substitution and dehydration

(4) Nucleophilic addition and dehydration

**NTA Ans. (3)**

**Sol.** Bakelite formation is example of electrophilic substitution and dehydration.



**19.** For the following Assertion and Reason, the correct option is

**Assertion** : For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7-9 elements.

**Reason** : The reactants are most strongly adsorbed on group 7-9 elements.

(1) Both assertion and reason are true but the reason is not the correct explanation for the assertion.

(2) Both assertion and reason are false.

(3) Both assertion and reason are true and the reason is the correct explanation for the assertion.

(4) The assertion is true, but the reason is false.

**NTA Ans. (4)**

**20.** The correct order of the calculated spin-only magnetic moments of complexes (A) to (D) is:

(A) $Ni(CO)_4$

(B) $[Ni(H_2O)_6]Cl_2$

(C) $Na_2[Ni(CN)_4]$

(D) $PdCl_2(PPh_3)_2$

(1) (A) ≈ (C) ≈ (D) < (B)

(2) (A) ≈ (C) < (B) ≈ (D)

(3) (C) < (D) < (B) < (A)

(4) (C) ≈ (D) < (B) < (A)

**NTA Ans. (1)**

**Sol.** $[Ni(CO)_4]$ $\mu_m$ = 0 B.M.

$[Ni(H_2O)_6]Cl_2$ $\mu_m$ = 2.8 B.M.

$Na_2[Ni(CN)_4]$ $\mu_m$ = 0 B.M.

$[PdCl_2(PPh_3)_2]$ $\mu_m$ = 0 B.M.

A ≈ C ≈ D < B

**21.** For an electrochemical cell

$Sn(s) \mid Sn^{2+}(aq, 1M) \| Pb^{2+}(aq, 1M) \mid Pb(s)$

the ratio $\dfrac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is ________.

(Given $E^0_{Sn^{2+}|Sn} = -0.14V$,

$E^0_{Pb^{2+}|Pb} = -0.13V, \dfrac{2.303RT}{F} = 0.06$)

**NTA Ans. (2.13 to 2.16)**

**Sol.** Cell reaction is :

$Sn(s) + Pb^{+2}(aq) \longrightarrow Sn^{+2}(aq) + Pb(s)$

Apply Nernst equation :

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]} \quad ...(1)$$

$$E^0_{cell} = -0.13 + 0.14 = 0.01 \text{ V}$$

At equilibrium : $E_{cell} = 0$

Substituting in (1)

$$0 = 0.01 - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{1}{3} = \log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{[Sn^{+2}]}{[Pb^{+2}]} = 2.15$$

**22.** At constant volume, 4 mol of an ideal gas when heated from 300 K to 500K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _______.

**NTA Ans. (6.25 to 6.25)**

**Sol.** For ideal gas :

$$\Delta U = nC_V[T_2 - T_1]$$

$$\Rightarrow \quad 5000 = 4 \times C_V[500 - 300]$$

$$\Rightarrow \quad C_V = \frac{5000}{800} = 6.25 \text{ J mole}^{-1} \text{ K}^{-1}$$

**23.** $NaClO_3$ is used, even in spacecrafts, to produce $O_2$. The daily consumption of pure $O_2$ by a person is 492L at 1 atm, 300K. How much amount of $NaClO_3$, in grams, is required to produce $O_2$ for the daily consumption of a person at 1 atm, 300 K ?

$NaClO_3(s) + Fe(s) \rightarrow O_2(g) + NaCl(s) + FeO(s)$

R = 0.082 L atm mol$^{-1}$ K$^{-1}$

**NTA Ans. (2120 to 2140)**

**Sol.** Mole of $O_2$ consumed $= \dfrac{1 \times 492}{0.082 \times 300} = 20$

Mole of $NaClO_3$ required = 20

Mass of $NaClO_3$ = 20 × 106.5 = 2130 gm

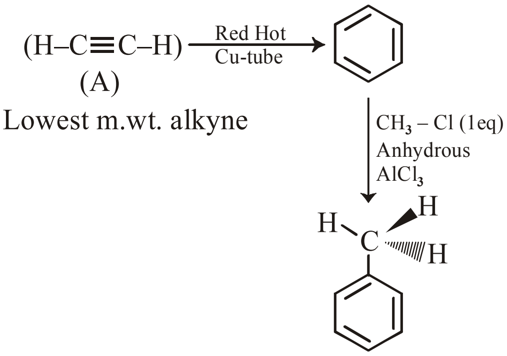
**24.** In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is _________.

$$A \xrightarrow[\text{Cu tube}]{\text{Red hot}} B \xrightarrow[\text{Anhydrous } AlCl_3]{CH_3Cl(1.eq.)} C$$

(A is a lowest molecular weight alkyne)

**NTA Ans. (13 to 13)**

**Sol.**




Total 13 atom are present in same plane (7 carbon & 6 hydrogen atoms. )

**25.** Complexes ($ML_5$) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal grometries, respectively. The sum of the 90°, 120° and 180° L-M L angles in the two complexes is ________.

**NTA Ans. (20 to 20)**

# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM**

## MATHEMATICS

## TEST PAPER WITH ANSWER & SOLUTION

**1.** Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is equal to

(1) $\frac{1}{2}$ (2) $-1$ (3) $-\frac{1}{2}$ (4) $-\frac{3}{2}$

**NTA Ans. (3)**

**Sol.** $\vec{b}\times\vec{c}-\vec{b}\times\vec{a}=\vec{0}$

$\vec{b}\times(\vec{c}-\vec{a})=\vec{0}$

$\vec{b}=\lambda(\vec{c}-\vec{a})$ ...(i)

$\vec{a}\cdot\vec{b}=\lambda\left(\vec{a}\cdot\vec{c}-\vec{a}^2\right)$

$4 = \lambda(0-6) \Rightarrow \lambda=\frac{-4}{6}=\frac{-2}{3}$

from (i) $\vec{b}=\frac{-2}{3}(\vec{c}-\vec{a})$

$\vec{c}=\frac{-3}{2}\vec{b}+\vec{a}=\frac{-1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

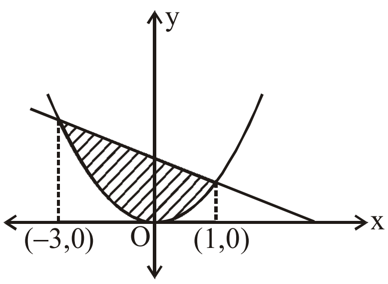
$\boxed{\vec{b}\cdot\vec{c}=\frac{-1}{2}}$ (3) Option

**2.** The area (in sq. units) of the region $\{(x,y)\in R^2 : x^2 \le y \le 3-2x\}$, is

(1) $\frac{29}{3}$ (2) $\frac{31}{3}$ (3) $\frac{34}{3}$ (4) $\frac{32}{3}$

**NTA Ans. (4)**

**Sol.**




Area = $\int_{-3}^{1}\left(3-2x-x^2\right)dx=\frac{32}{3}$

(4) option

**3.** The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$ (3) 2 (4) $\sqrt{2}$

**NTA Ans. (2)**

**Sol.** $x^2 + 2xy - 3y^2 = 0$

$m_N$ = slope of normal drawn to curve at (2,2) is $-1$

L : x + y = 4.

perpendicular distance of L from (0,0)

$=\frac{|0+0-4|}{\sqrt{2}}=2\sqrt{2}$

(2) Option

**4.** If $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then :

(1) $\frac{1}{9}<I^2<\frac{1}{8}$ (2) $\frac{1}{16}<I^2<\frac{1}{9}$

(3) $\frac{1}{6}<I^2<\frac{1}{2}$ (4) $\frac{1}{8}<I^2<\frac{1}{4}$

**NTA Ans. (1)**

**Sol.** $f(x)=\frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$f'(x)=\frac{-6(x-1)(x-2)}{2\left(2x^3-9x^2+12x+4\right)^{3/2}}$

$\therefore f(x)$ is decreasing in (1,2)

$f(1)=\frac{1}{3};\ f(2)=\frac{1}{\sqrt{8}}$

$\frac{1}{3}<I<\frac{1}{\sqrt{8}} \Rightarrow I^2\in\left(\frac{1}{9},\frac{1}{8}\right)$

(1) Option

**5.** If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line $L_1$, where $L_1$ is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

(1) $c^2 - 6c + 7 = 0$

(2) $c^2 + 6c + 7 = 0$

(3) $c^2 + 7c + 6 = 0$

(4) $c^2 - 7c + 6 = 0$

**NTA Ans. (2)**

**Sol.** Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow m_T|_P = -1$

$y = mx + c$ is tangent to $(x-3)^2 + y^2 = 1$

$y = x + c$ is tangent to $(x-3)^2 + y^2 = 1$

$\left|\frac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

**6.** Let S be the set of all functions $f : [0,1] \to R$, which are continuous on [0,1] and differentiable on (0,1). Then for every $f$ in S, there exists a $c \in (0,1)$, depending on $f$, such that

(1) $|f(c) - f(1)| < (1 - c)|f'(c)|$

(2) $|f(c) - f(1)| < |f'(c)|$

(3) $|f(c) + f(1)| < (1 + c)|f'(c)|$

(4) $\frac{f(1) - f(c)}{1 - c} = f'(c)$

**NTA Ans. (2)**

**Sol. Bonus**

option (1), (2), (3) are incorrect for $f(x)$ = constant and option (4) is incorrect

$\frac{f(1) - f(c)}{1 - c} = f'(a)$ where $c < a < 1$ (use LMVT)

Also for $f(x) = x^2$ option (4) is incorrect.

**7.** Which of the following statements is a tautology?

(1) $\sim(p \vee \sim q) \to p \vee q$

(2) $\sim(p \wedge \sim q) \to p \vee q$

(3) $\sim(p \vee \sim q) \to p \wedge q$

(4) $p \vee (\sim q) \to p \wedge q$

**NTA Ans. (1)**

**Sol.** $\sim(p \vee \sim q) \to p \vee q$

$(\sim p \wedge q) \to p \vee q$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

(1) Option

**8.** If the 10[th] term of an A.P. is $\frac{1}{20}$ and its 20[th] term is $\frac{1}{10}$, then the sum of its first 200 terms is

(1) $50\frac{1}{4}$ (2) $100\frac{1}{2}$

(3) 50 (4) 100

**NTA Ans. (2)**

**Sol.** $T_{10} = \frac{1}{20} = a + 9d$ ...(i)

$T_{20} = \frac{1}{10} = a + 19d$ ...(ii)

$a = \frac{1}{200} = d$

Hence, $S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2}$

(2) Option

**9.** Let $f : (1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1 + x^2}$, where [x] denotes the greatest integer $\le x$. Then the range of $f$ is

(1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

(3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

**NTA Ans. (4)**

**Sol.** $f(x) = \begin{cases} \dfrac{x}{x^2+1} & ; \quad x \in (1,2) \\ \dfrac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$

$f(x)$ is decreasing function

$\therefore \ f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(4) Option

**10.** The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has

(1) infinitely many solutions when $\lambda = 2$

(2) a unique solution when $\lambda = -8$

(3) no solution when $\lambda = 8$

(4) no solution when $\lambda = 2$

**NTA Ans. (4)**

**Sol.** $D = \begin{vmatrix} \lambda & 3 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda+8)(2-\lambda)$

for $\lambda = 2$ ; $D_1 \neq 0$

Hence, no solution for $\lambda = 2$

(4) Option

**11.** If $\alpha$ and $\beta$ be the coefficients of $x^4$ and $x^2$ respectively in the expansion of

$\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6$, then

(1) $\alpha + \beta = 60$ (2) $\alpha + \beta = -30$

(3) $\alpha - \beta = -132$ (4) $\alpha - \beta = 60$

**NTA Ans. (3)**

**Sol.** $2[{}^6C_0 . x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$\alpha = -96$ & $\beta = 36$

$\therefore \ \alpha - \beta = -132$

(3) Option

**12.** $\lim\limits_{x\to 0} \dfrac{\int_0^x t\sin(10t)\,dt}{x}$ is equal to

(1) 0 (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$ (4) $\frac{1}{10}$

**NTA Ans. (1)**

**Sol.** Using L.H. Rule

$\lim\limits_{x\to 0} \dfrac{x\sin(10x)}{1} = 0$

(1) Option

**13.** If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to

(1) $4I - A$ (2) $A - 6I$

(3) $6I - A$ (4) $A - 4I$

**NTA Ans. (2)**

**Sol.** $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$; $|A| = 8 - 18 = -10$

$A^{-1} = \dfrac{\text{adj}A}{|A|} = \dfrac{\begin{pmatrix} 4 & -2 \\ -9 & 2 \end{pmatrix}}{-10}$

$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I$

(2) Option

**14.** The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

(1) 3.99 (2) 3.98

(3) 4.02 (4) 4.01

**NTA Ans. (1)**

**Sol.** $\dfrac{\sum x_i}{20} = 10 \Rightarrow \Sigma x_i = 200$ ...(i)

$$\frac{\sum x_i^2}{20} - 100 = 4 \Rightarrow \Sigma x_i^2 = 2080 \quad ...(ii)$$

Actual mean = $\frac{200-9+11}{20} = \frac{202}{20}$

Variance = $\frac{2080-81+121}{20} - \left(\frac{202}{20}\right)^2 = 3.99$

(1) Option

**15.** If a hyperbola passes through the point P(10,16) and it has vertices at (±6,0), then the equation of the normal to it at P is

(1) x + 2y = 42 (2) 3x + 4y = 94

(3) 2x + 5y = 100 (4) x + 3y = 58

**NTA Ans. (3)**

**Sol.** $\frac{x^2}{36} - \frac{y^2}{b^2} = 1 \quad ...(i)$

P(10,16) lies on (i) get $b^2 = 144$

$\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is

$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2$

2x + 5y = 100

(3) Option

**16.** Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

(1) 0.02 (2) 0.01 (3) 0.20 (4) 0.10

**NTA Ans. (4)**

**Sol.** $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{10}$

(4) Option

**17.** The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane ?

(1) (–1, –1, –1) (2) (–1, –1, 1)

(3) (1, 1, 1) (4) (1, –1, 1)

**NTA Ans. (4)**

**Sol.** Point on plane $R\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k}$

Equation of require plane is x + y + z = 1

Hence (1, –1, 1) lies on plane

(4) Option

**18.** Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) is an empty set.

(2) contains at least four elements.

(3) contains exactly two elements.

(4) is a singleton.

**NTA Ans. (4)**

**Sol.** Let $3^x = t$ ; t > 0

$t(t - 1) + 2 = |t - 1| + |t - 2|$

$t^2 - t + 2 = |t - 1| + |t - 2|$

**Case-I :** t < 1

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + 2 = 3 - t$

$t^2 + t - 1 = 0$

$$t = \frac{-1 \pm \sqrt{5}}{2}$$

$t = \dfrac{\sqrt{5}-1}{2}$ is only acceptable

**Case-II :** $1 \le t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

$D < 0$ no real solution

**Case-III :** $t \ge 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t \quad 5 = 0 \Rightarrow D < 0$ no real solution

(4) Option

**19.** Let $\alpha = \dfrac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation :

(1) $x^2 - 102x + 101 = 0$

(2) $x^2 + 101x + 100 = 0$

(3) $x^2 - 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

**NTA Ans. (1)**

**Sol.** $\alpha = \omega$

$a = (1 + \omega)(1 + \omega^2 + \omega^4 + ..... + \omega^{200})$

$$a = (1+\omega)\frac{\left(1-\left(\omega^2\right)^{101}\right)}{1-\omega^2} = 1$$

$b = 1 + \omega^3 + \omega^6 + ..... + \omega^{300} = 101$

$x^2 - 102x + 101 = 0$

(1) Option

**20.** The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is

(1) $x(y')^2 = x + 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $xy'' = y'$

(4) $x(y')^2 = x - 2yy'$

**NTA Ans. (1)**

**Sol.** $2x = 4by' \Rightarrow y' = \dfrac{2x}{4b}$

Required D.E. is $x^2 = \dfrac{2x}{y'}y + \left(\dfrac{x}{y'}\right)^2$

$x(y')^2 = 2yy' + x$

(1) Option

**21.** If $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \dfrac{1}{7}$ and $\sqrt{\dfrac{1-\cos 2\beta}{2}} = \dfrac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \dfrac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

**NTA Ans. (1)**

**Sol.** $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \dfrac{1}{7} \Rightarrow \tan\alpha = \dfrac{1}{7}$

$$\sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3} \Rightarrow \tan 2\beta = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \tan 2\beta} = 1$$

Ans. 1.00

**22.** Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at $x$ = _______.

**NTA Ans. (3)**

**Sol.** $f''(x) = \lambda(x - 1)$

$$f'(x) = \frac{\lambda x^2}{2} - \lambda x + C \Rightarrow f'(-1) = 0 \Rightarrow c = \frac{-3\lambda}{2}$$

$$f(x) = \frac{\lambda x^3}{6} - \frac{\lambda x^2}{2} - \frac{3\lambda}{2}x + d$$

$f(1) = -6 \Rightarrow -11\lambda + 6d = -36$ ...(i)

$f(-1) = 10 \Rightarrow 5\lambda + 6d = 60$ ...(ii)

from (i) & (ii) $\lambda = 6$ & $d = 5$

$f(x) = x^3 - 3x^2 - 9x + 5$

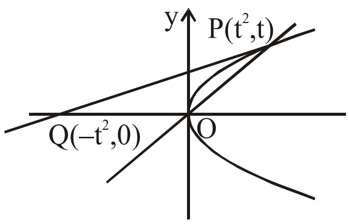
Which has minima at $x = 3$

Ans. 3.00

**23.** Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to ___________.

**NTA Ans. (0.50)**

**Sol.** $\Delta OPQ = 4$




$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

$t = 2$ $(\because t > 0)$

$$\therefore \ m = \frac{1}{2}$$

Ans. 0.50

**24.** The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to ________.

**NTA Ans. (504)**

**Sol.** $\frac{1}{4}\left(\sum_{n=1}^{7} 2n^3 + \sum_{n=1}^{7} 3n^2 + \sum_{n=1}^{7} n\right)$

$$= \frac{1}{4}\left(2\left(\frac{7\times 8}{2}\right)^2 + 3\left(\frac{7\times 8\times 15}{6}\right) + \frac{7\times 8}{2}\right)$$

$= 504$

Ans. 504.00

**25.** The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is ________.

**NTA Ans. (2454)**

**Sol.** $N \to 2$, $A \to 2$, $I \to 2$, E, X, M, T, $O \to 1$

| Category | Selection | Arrangement |
|---|---|---|
| 2 alike of one kind and 2 alike of other kind | ${}^3C_2 = 3$ | $3 \times \frac{4!}{2!\,2!} = 18$ |
| 2 alike and 2 different | ${}^3C_1 \times {}^7C_2$ | ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$ |
| All 4 different | ${}^8C_4$ | ${}^8C_4 \times 4! = 1680$ |

Total = 2454

Ans. 2454.00