FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

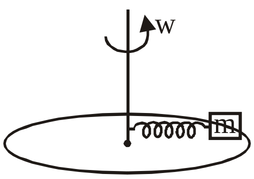
1. A spring mass system (mass m, spring constant k and natural length *l*) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, (k >> mω²) the relative change in the length of the spring is best given by the option :

(1) $\frac{2m\omega^2}{k}$ (2) $\frac{m\omega^2}{3k}$

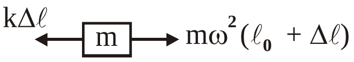
(3) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (4) $\frac{m\omega^2}{k}$

NTA Ans. (4)

Sol.



FBD of m in frame of disc/-



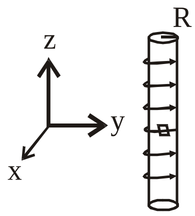
$k\Delta\ell = m\omega^2(\ell_0 + \Delta\ell)$

$$\Delta\ell = \frac{m\omega^2\ell_0}{k - m\omega^2} \approx \frac{m\omega\ell_0}{k}$$

$$\frac{\Delta\ell}{\ell_0} = \text{Relative change} = \frac{m\omega^2}{k}$$

∴ Correct answer (4)

2. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning) :

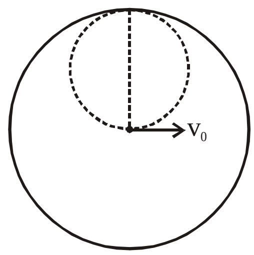


(1) $\frac{e\mu_0 nIR}{m}$ (2) $\frac{e\mu_0 nIR}{2m}$

(3) $\frac{2e\mu_0 nIR}{m}$ (4) $\frac{e\mu_0 nIR}{4m}$

NTA Ans. (2)

Sol. Top view of solenoid



Maximum possible radius of electron = $\frac{R}{2}$

$$\therefore \frac{R}{2} = \frac{mv}{qB} = \frac{mv_{max}}{e(\mu_0 ni)}$$

$$v_{max} = \frac{R}{2}\frac{e\mu_0 ni}{m}$$

∴ Correct answer = 2

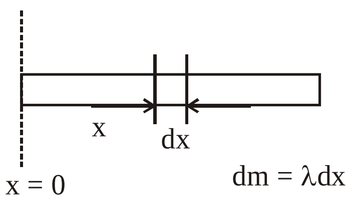
3. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at :

(1) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (2) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

(3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (4) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

NTA Ans. (4)

Sol.



$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int(\lambda dx)x}{\int dm}$$

$$= \frac{\int_0^L\left(a + \frac{bx^2}{L^2}\right)x\,dx}{\int_0^L\left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{b}{L^2}\cdot\frac{L^4}{4}}{aL + \frac{b}{L^2}\cdot\frac{L^3}{3}}$$

$$= \frac{\left(\frac{4a+2b}{8}\right)L}{\frac{(3a+b)}{3}} = \frac{3}{4}\frac{(2a+b)L}{(3a+b)}$$

$\therefore$ correct answer 4

4. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) :

(1) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(3) $B_0\hat{k}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(4) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t + k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

NTA Ans. (1)

Sol. Direction of polarisation $= \hat{E} = \hat{k}$

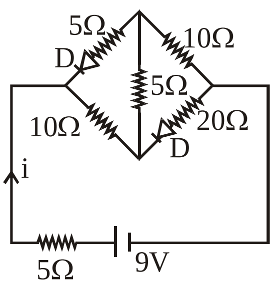
Direction of propagation $= \hat{E}\times\hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore \hat{E}\times\hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\hat{B} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

Correct answer (1)

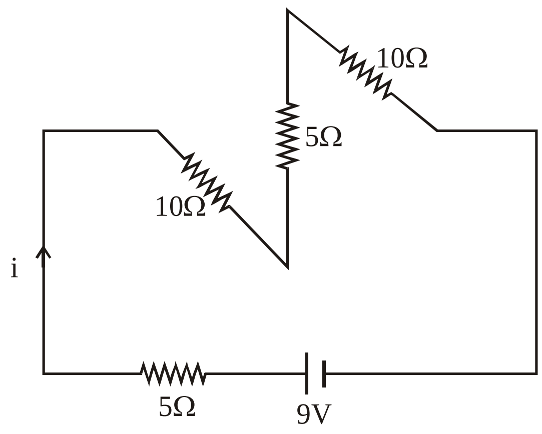
5. The current i in the network is :



(1) 0 A (2) 0.6 A
(3) 0.3 A (4) 0.2 A

NTA Ans. (3)

Sol.



$$i=\frac{9}{(5+10+5+10)}=\frac{9}{30}A$$

∴ Correct answer (3)

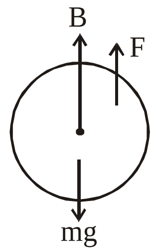
6. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) :

(1) $r=\sqrt{\frac{2T}{3(d+\rho)g}}$ (2) $r=\sqrt{\frac{3T}{(2d-\rho)g}}$

(3) $r=\sqrt{\frac{T}{(d-\rho)g}}$ (4) $r=\sqrt{\frac{T}{(d+\rho)g}}$

NTA Ans. (2)

Sol. FBD of droplet



B + F = mg

$$B=\left(\frac{2}{3}\pi R^3\right)\rho g$$

F = T(2πR)

$$m=d\left(\frac{4}{3}\pi R^3\right)$$

$$\left(\frac{2}{3}\pi R^3\right)\rho g+T(2\pi R)=d\left(\frac{4}{3}\pi R^3\right)g$$

$$T(2\pi R)=\left(\frac{2}{3}\pi R^3\right)g[2d-\rho]$$

$$R=\sqrt{\frac{3T}{(2d-\rho)g}}$$

∴ Correct answer (2)

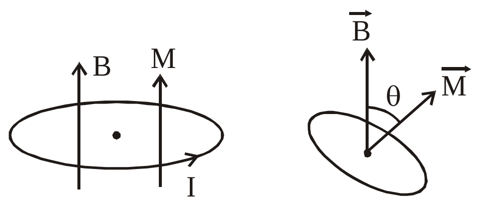
7. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then :

(1) $T=\sqrt{\frac{\pi m}{2IB}}$ (2) $T=\sqrt{\frac{2\pi m}{IB}}$

(3) $T=\sqrt{\frac{\pi m}{IB}}$ (4) $T=\sqrt{\frac{2m}{IB}}$

NTA Ans. (2)

Sol.



$$\vec{T}=\vec{M}\times\vec{B}=-MB\sin\theta$$

$I\alpha = -MB\sin\theta$

for small θ,

$$\alpha = -\frac{MB}{I}\theta$$

$$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{(i)(\pi R^2)B}{\left(\frac{mR^2}{2}\right)}}$$

$$\omega = \sqrt{\frac{2i\pi B}{m}}$$

$$\therefore = T = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi m}{iB}}$$

$\therefore$ Correct answer (2)

8. A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are : 420 Hz and 490 Hz. Then L in meters is :

(1) 8.1 m (2) 5.1 m (3) 1.1 m (4) 2.1 m

NTA Ans. (4)

Sol. $\frac{nv}{2\ell} = 420$

$$\frac{(n+1)v}{2\ell} = 490$$

$$\frac{v}{2\ell} = 70$$

$$\ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}} = \frac{1}{140}\sqrt{90\times10^3}$$

$$\ell = \frac{300}{140} = 2.142$$

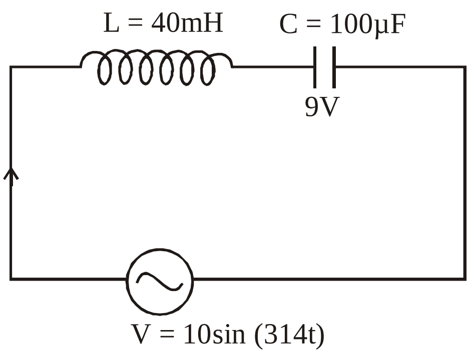
$\therefore$ Correct answer (4)

9. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10sin(314 t) is applied to the circuit, the current in the circuit is given as :

(1) 0.52 cos 314 t (2) 0.52 sin 314 t
(3) 10 cos 314 t (4) 5.2 cos 314 t

NTA Ans. (1)

Sol.

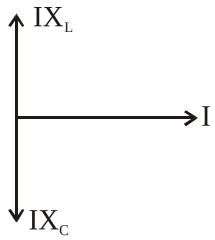


$X_L = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{314\times100\times10^{-6}}$$

$$= \frac{10^4}{314} = 31.84\Omega$$

Phasor



$V_m = I_m(X_C - X_L)$
$10 = I_m(31.84 - 12.56)$

$$I_m = \frac{10}{19.28} = 0.52A$$

$$I = 0.52 \sin\left(314t + \frac{\pi}{2}\right)$$

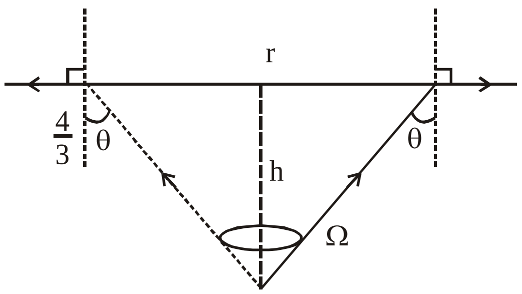
$\therefore$ Correct answer (1)

10. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) : [Use the fact that surface area of a spherical cap of height h and radius of curvature r is 2πrh]:

(1) 17% (2) 21%
(3) 34% (4) 50%

NTA Ans. (1)

Sol.



$$\frac{4}{3}\sin\theta = 1\sin 90°$$

$$\sin\theta = \frac{3}{4}$$

Area of sphere in which light spread = $4\pi R^2$

$\Omega = 2\pi\ (1 - \cos\ \theta)$

$$\Omega = 2\pi\left(1 - \frac{\sqrt{7}}{4}\right)$$

P $\rightarrow$ 4π steradians

$$P' \rightarrow \frac{P}{4\pi}(1-\cos\theta)$$

$$\text{Ratio} = \frac{P'}{P} = \frac{2\pi(1-\cos\theta)}{4\pi} = \frac{(1-\cos\theta)}{2} = \frac{1-\frac{\sqrt{7}}{4}}{2}$$

$$= \frac{0.33}{2} = 0.17$$

$\therefore$ Correct answer (1)

11. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The raito of their respective mean free times is closest to :

(1) 3.67 (2) 4.67

(3) 1.83 (4) 2.3

NTA Ans. (1)

Sol. $$\lambda = \frac{1}{\sqrt{2}\pi n_v d^2}$$

$$\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v} = \frac{1}{\sqrt{2}\pi n_v d^2}\sqrt{\frac{M}{3RT}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}}\frac{d_2^2}{d_1^2}$$

$$= \sqrt{\frac{40}{140}}\frac{(0.1)^2}{(0.07)^2}$$

= 1.09

$\therefore$ Nearest possible answer (3)

12. A particle starts from the origin at t = 0 with an initial velocity of $3.0\,\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})\,m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is :-

(1) 50 (2) 32 (3) 60 (4) 40

NTA Ans. (3)

Sol. $$x = u_x t + \frac{1}{2}a_x t^2$$

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$32 = 0 \times t + \frac{1}{2}(4)(t)^2$$

$t^2 = 16$

t = 4 sec

$$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$$

= 12 + 48 = 60 m

$\therefore$ Correct answer (3)

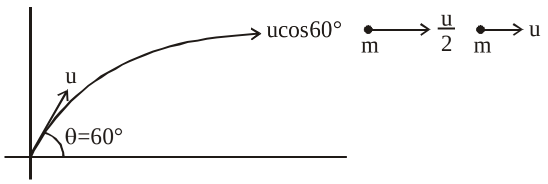
13. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

(1) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ (2) $2\sqrt{2}\frac{u^2}{g}$

(3) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (4) $\frac{5}{8}\frac{u^2}{g}$

NTA Ans. (3)

Sol.



By momentum conservation,

$$\frac{mu}{2} + mu = 2mv'$$

$$v' = \frac{3v}{4}$$

Range after collision = $\frac{3v}{4}\sqrt{\frac{2H}{g}}$

$$= \frac{3v}{4}\sqrt{\frac{2 \cdot u^2 \sin^2 60^\circ}{g\,2g}}$$

$$= \frac{3}{4}\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

∴ Correct answer (3)

14. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ?

(1) 35.8 nm

(2) 24.2 nm

(3) 8.6 nm

(4) 11.4 nm

NTA Ans. (4)

Sol. 1 Rydberg energy = 13.6 eV

So, ionisation energy = $(13.6\ Z^2)$eV

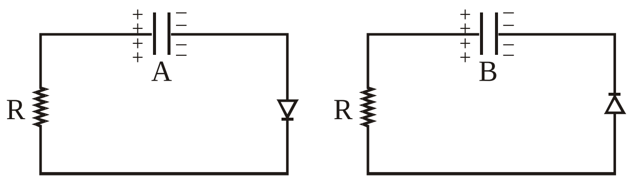
= 9 × 13.6eV

Z = 3

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.09 \times 10^7 \times 9 \times \frac{8}{9}$$

λ = 11.4 nm

15. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)



(1) $Q_A = VC, Q_B = \frac{VC}{e}$

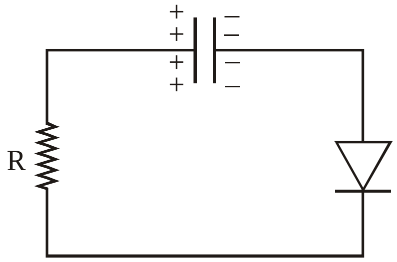
(2) $Q_A = \frac{CV}{2}, Q_B = \frac{VC}{e}$

(3) $Q_A = VC,\ Q_B = CV$

(4) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$

NTA Ans. (1)

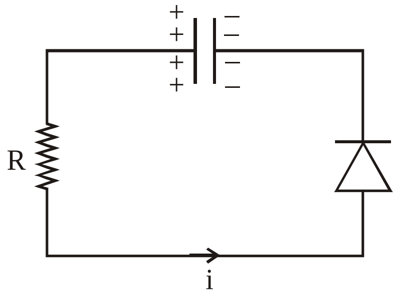
Sol. For (A)



No current flows

Hence $Q_A = CV$

For (B)



$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

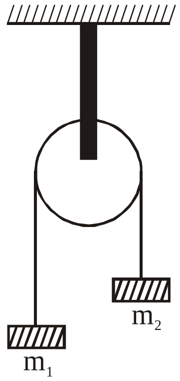
$$q = CVe^{-\frac{t}{RC}}$$

at t = CR

$$Q_B = CVe^{-1} = \frac{CV}{e}$$

∴ Correct answer (1)

16. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is :



(1) $\left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

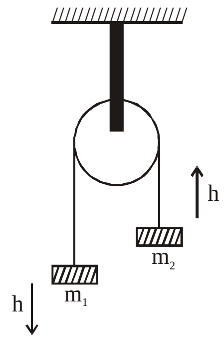
(2) $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(4) $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

NTA Ans. (2)

Sol.



by using work energy theorem

Wg = ΔKE

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)V^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$$

$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

∴ Correct answer (2)

17. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are ν_A and ν_B, respectively, then $\frac{\nu_A}{\nu_B} = \frac{n}{4}$.

The value of n is :

(1) 4 (2) 1 (3) 2 (4) 3

NTA Ans. (1)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$ (Escape velocity)

$$V_A = \sqrt{\frac{2GM}{R}}$$

$$V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$$

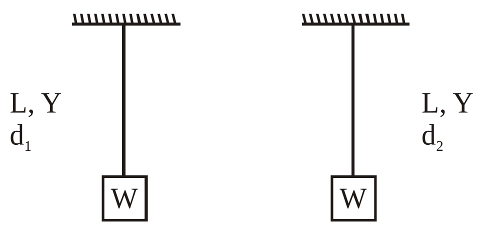
$\therefore$ Correct answer (1)

18. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is:

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) 2 : 1 (4) $\sqrt{2}:1$

NTA Ans. (4)

Sol.



$$\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$$

$$\frac{u_1}{u_2} = \frac{1}{4} \Rightarrow 4u_1 = u_2$$

$$4\frac{1}{2Y}\left[\frac{W\cdot 4}{\pi d_1^2}\right]^2 = \frac{1}{2Y}\left[\frac{W\cdot 4}{\pi d_2^2}\right]^2$$

$$4 = \left(\frac{d_1}{d_2}\right)^4$$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{2}:1$$

$\therefore$ Correct answer (4)

19. For the four sets of three measured physical quantities as given below. Which of the following options is correct ?

(i) $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$

(ii) $A_2 = 24.44$, $B_2 = 16.082$, $C_2 = 240.2$

(iii) $A_3 = 25.2$, $B_3 = 19.2812$, $C_3 = 236.183$

(iv) $A_4 = 25$, $B_4 = 236.191$, $C_4 = 19.5$

(1) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

(2) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

(3) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

(4) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$

NTA Ans. (4)

Sol. $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2$

= 280.6324

= 280.6 (After rounding off)

$A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2$

= 280.722

= 280.7 (After rounding off)

$A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183$

= 280.6642

= 280.7 (After rounding off)

$A_4 + B_4 + C_4 = 25 + 236.191 + 19.5$

= 280.691

= 281 (After rounding off)

$A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

No option is matching Question should be (BONUS)

Best possible option is (2)

∴ Correct answer (2)

20. An electron of mass m and magnitude of charge |e| initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is :

(1) $\frac{-h}{|e|Et^2}$ (2) $\frac{|e|Et}{h}$

(3) $-\frac{h}{|e|E\sqrt{t}}$ (4) $-\frac{h}{|e|Et}$

NTA Ans. (1)

Sol. $a = \frac{eE}{m}$

$v = u + at = \left(\frac{eE}{m}\right)t$

$\lambda = \frac{h}{mv}$

$\frac{d\lambda}{dt} = \frac{-(hm)\cdot\frac{dv}{dt}}{(mv)^2} = -\frac{ah}{mv^2} = -\frac{h}{|e|Et^2}$

∴ Correct answer (1)

21. Starling at temperature 300 K, one mole of an ideal diatomic gas (γ = 1.4) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ________.

NTA Ans. (1818.00)

Sol. PV^{γ} = constant

$TV^{\gamma-1} = C$

$300 \times V^{\frac{7}{5}-1} = T_2\left(\frac{V}{16}\right)^{\frac{7}{5}-1}$

$300 \times 2^{4\times\frac{2}{5}} = T_2$

Isobaric process

$V = \frac{nRT}{P}$

$V_2 = kT_2$ ... (1)

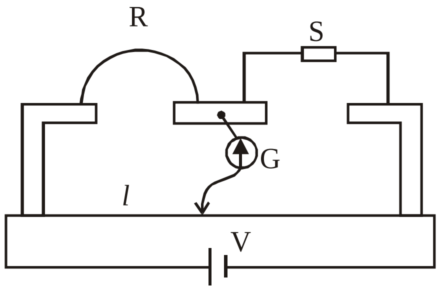
$2V_2 = KT_f$ ... (2)

$\frac{1}{2} = \frac{T_2}{T_f} \Rightarrow T_f = 2T_2$

$T_f = 2 \times 300 \times 2^{\frac{8}{5}}$ = 1818.85

∴ Correct answer 1819

22. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It Is found that balancing length is l = 25 cm. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance l' (in cm) will now be________.



NTA Ans. (40.00)

Sol. In balancing

$$\frac{R}{S}=\frac{25}{75}$$

New resistance $R'=\frac{\rho\ell}{A}$

$$=\frac{\rho\times\frac{\ell}{2}}{\frac{A}{4}}=\frac{\rho\ell}{2}\times 4A$$

R' = 2R

$$\frac{2R}{S}=\frac{\ell'}{100-\ell'}$$

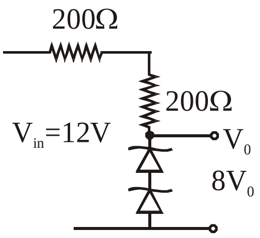
$$2\times\frac{1}{3}=\frac{\ell'}{100-\ell'}=3\ell'=200-2\ell'$$

$$5\ell'=200$$

$$\ell'=40$$

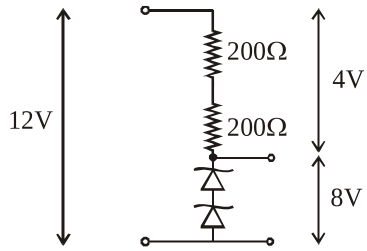
∴ Correct answer 40

23. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both zener diodes are identical) _________.



NTA Ans. (12.00)

Sol.



Current in circuit = $\frac{4}{400}=\frac{1}{100}A$

So power dissipited in each diode = VI

$$=4\times\frac{1}{100}W$$

= 40×10^{-3} mW

∴ Correct answer 40

24. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) ___________.

NTA Ans. **(750.00)**

Sol. The length of the screen used portion for 15 fringes, and also for ten fringes

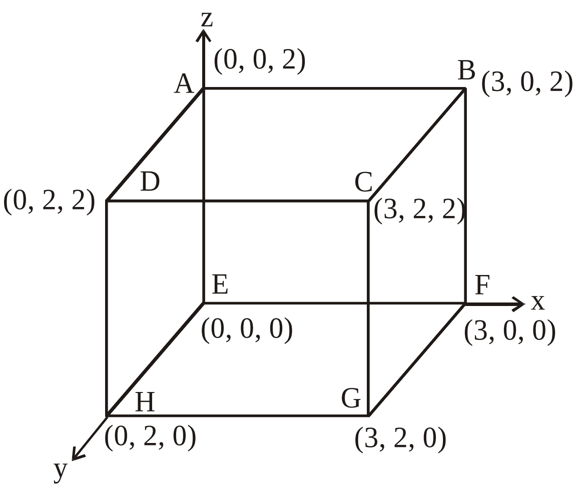
$$15 \times 500 \times \frac{D}{\lambda} = 10 \times \frac{\lambda D}{\lambda}$$

$15 \times 50 = \lambda$

$\lambda = 750$ nm

$\therefore$ Correct answer 750

25. An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}\,N/C$ passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm^2/C) ____________.



NTA Ans. **(48.00)**

Sol. The flux passes through ABCD (x – y) plane is zero, because electric field parallel to surface. Flux of the electric field through surface BCGF (y - z)

At BCGF (electric field) $\Rightarrow \vec{E} = 12\hat{i} - \left(y^2 + 1\right)\hat{j}$

(x = 3m)

Flux $\phi_{II} = 12 \times 4 = 48\ Nm^2/C$

So $\phi_I - \phi_{II} = 0 - 48 = -48\ Nm^2/C$

$\therefore$ Correct answer –48

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. The correct order of the spin-only magnetic moments of the following complexes is :

(I) $[Cr(H_2O)_6]Br_2$

(II) $Na_4[Fe(CN)_6]$

(III) $Na_3[Fe(C_2O_4)_3]$ $(\Delta_0 > P)$

(IV) $(Et_4N)_2[CoCl_4]$

(1) (III) > (I) > (II) > (IV)

(2) (I) > (IV) > (III) > (II)

(3) (II) ≈ (I) > (IV) > (III)

(4) (III) > (I) > (IV) > (II)

NTA Ans. (2)

Sol. I $\left[Cr(H_2O)_6\right]^{2+}$

$Cr^{+2} \Rightarrow [Ar]\,3d^4$

$H_2O \rightarrow$ Weak field ligand

[↑][ ] e_g

[↑][↑][↑] t_{2g}

Unpaired e^- = 4

Magnetic moment = $\sqrt{24}$ BM
= 4.89 BM

II $\left[Fe(CN)_6\right]^{4-}$

$Fe^{+2} \Rightarrow [Ar]\,3d^6$

$CN^- \rightarrow$ Strong field ligand

[ ][ ] e_g

[↑↓][↑↓][↑↓] t_{2g}

Unpaired e^- = 0

Magnetic moment = 0 BM
= 0 BM

III $\left[Fe(C_2O_4)_3\right]^{3-}$

$Fe^{+3} \Rightarrow [Ar]\,3d^5$

As $\Delta_0 > P$

[ ][ ] e_g

[↑↓][↑↓][↑] t_{2g}

Unpaired e^- = 1

Magnetic moment = $\sqrt{3}$ BM
= 1.73 BM

IV $\left(Et_4N\right)^+\left[CoCl_4\right]^{2-}$

$Co^{+2} \Rightarrow [Ar]\,3d^7$

$\left[CoCl_4\right]^{-2}$ = [↑][↑][↑] t_{2g}

[↑↓][↑↓] e_g

Unpaired electrons = 3

Magnetic moment = $\sqrt{15}$ BM
= 3.87 BM

Hence order of magnetic moment is
I > IV > III > II

2. The first and second ionisation enthalpies of a metal are 496 and 4560 kJ mol^{-1}, respectively. How many moles of HCl and H_2SO_4, respectively, will be needed to react completely with 1 mole of the metal hydroxide ?

(1) 1 and 0.5 (2) 2 and 0.5

(3) 1 and 1 (4) 1 and 2

NTA Ans. (1)

Sol. IE values indicate, that the metal belongs to Ist group since second IE is very high
($\because$ only one valence electron)
Metal hydroxide will be of type, MOH.

$MOH + HCl \rightarrow MCl + H_2O$
(1mol) (1mol)

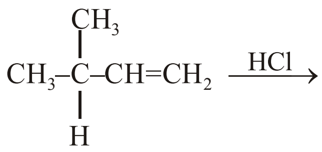
$MOH + \frac{1}{2}H_2SO_4 \rightarrow \frac{1}{2}M_2SO_4 + H_2O$

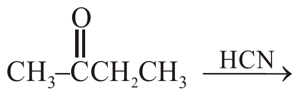
(1mol) ($\frac{1}{2}$ mol)

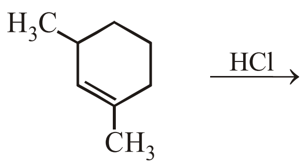
So one mole of HCl required to react with one mole MOH.

So $\frac{1}{2}$ mole of H_2SO_4 required to react with one mole MOH.

3. Which of the following reactions will not produce a racemic product ?

(1) 

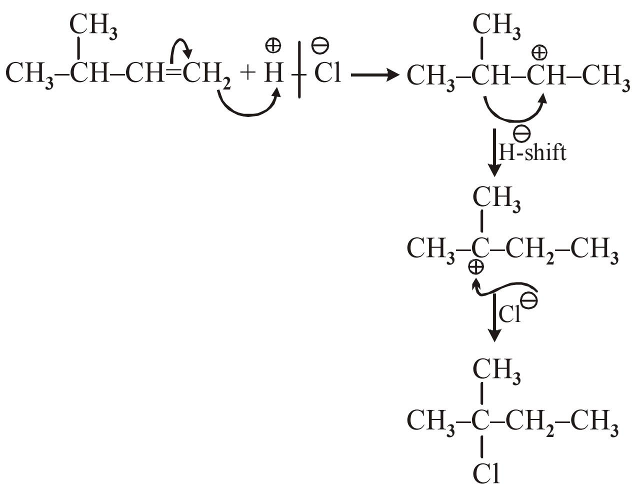
(2) 

(3) 

(4) $CH_3CH_2CH=CH_2 \xrightarrow{HBr}$

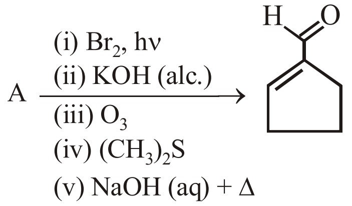
NTA Ans. (1)

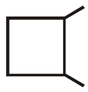
Sol.

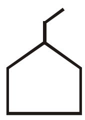


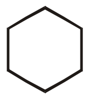
(No chiral centre, so no racemisation possible)

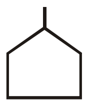
4. In the following reaction A is :



(1) 

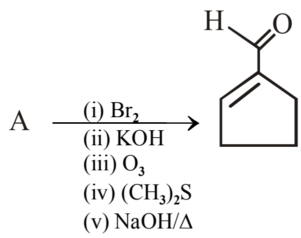
(2) 

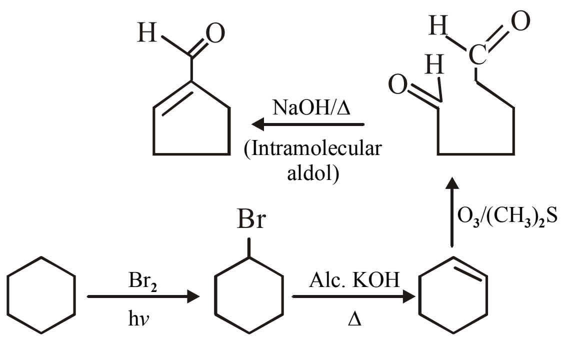
(3) 

(4) 

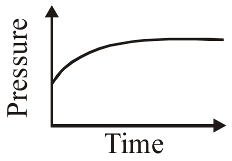
NTA Ans. (3)

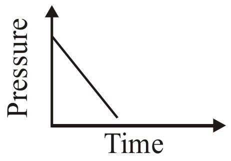
Sol.

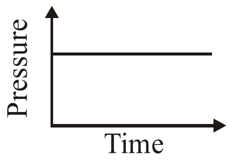


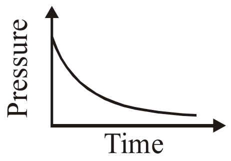


5. A mixture of gases O_2, H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behaviour of pressure with time is :

(1) 

(2) 

(3) 

(4) 

NTA Ans. (4)

Sol. Adsorption of Gases will decreases

6. Which polymer has 'chiral' monomer(s) ?

(1) Buna-N (2) Nylon 6,6

(3) Neoprene (4) PHBV

NTA Ans. (4)

Sol. PHBV :

Poly β-hydroxy butyrate-co-β-hydroxy valerate

$CH_3–CH(OH)–CH_2–COOH$

(3-hydroxy butanoic acid)

+

$CH_3–CH_2–CH(OH)–CH_2–C(=O)–OH$

(3-hydroxy pentanoic acid)

7. Biochemical Oxygen Demand (BOD) is the amount of oxygen required (in ppm):

(1) by anaerobic bacteria to breakdown inorganic waste present in a water body.

(2) for the photochemical breakdown of waste present in 1 m^3 volume of a water body.

(3) by bacteria to break-down organic waste in a certain volume of a water sample.

(4) for sustaining life in a water body.

NTA Ans. (3)

Sol. Biochemical oxygen demand (BOD) is amount of oxygen required by bacteria to break down organic waste in a certain volume of water sample.

8. Among the statements (a)-(d) the correct ones are:

(a) Lithium has the highest hydration enthalpy among the alkali metals.

(b) Lithium chloride is insoluble in pyridine.

(c) Lithium cannot form ethynide upon its reaction with ethyne.

(d) Both lithium and magnesium react slowly with H_2O.

(1) (a), (b) and (d) only

(2) (b) and (c) only

(3) (a), (c) and (d) only

(4) (a) and (d) only

NTA Ans. (3)

Sol. Lithium has highest hydration enthalpy among alkali metals due to its small size.

LiCl is soluble in pyridine because LiCl have more covalent character.

Li does not form ethynide with ethyne.

Both Li and Mg reacts slowly with H_2O

9. Amongst the following, the form of water with the lowest ionic conductance at 298 K is:

(1) distilled water

(2) water from a well

(3) saline water used for intravenous injection

(4) sea water

NTA Ans. (1)

Sol. Distilled water have lowest ionic conductance.

10. Which of the following has the shortest C-Cl bond?

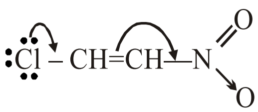
(1) Cl–CH=CH–OCH_3

(2) C1–CH=CH–CH_3

(3) C1–CH=CH_2

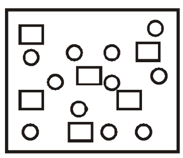
(4) C1–CH=CH–NO_2

NTA Ans. (4)

Sol. 

Due to –M effect of –NO_2 and + M effect of Cl more D.B. character between C – Cl bond. So shortest bond length.

11. In the figure shown below reactant A (represented by square) is in equilibrium with product B (represented by circle). The equilibrium constant is :



(1) 2 (2) 1 (3) 8 (4) 4

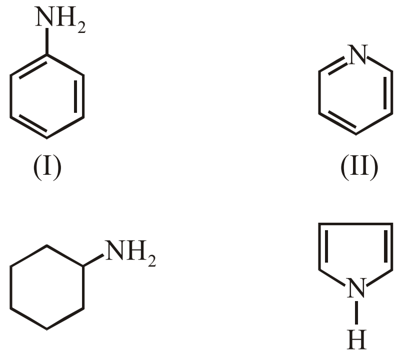
NTA Ans. (1)

Sol. Bonus (no reaction is given)

$A \rightleftharpoons B$ (Assume reaction)

$$K = \frac{[B]}{[A]} = \frac{11}{6} \simeq 2$$

12. The decreasing order of basicity of the following amines is :



(I) (II) (III) (IV)

(1) (I) > (III) > (IV) > (II)

(2) (III) > (I) > (II) > (IV)

(3) (III) > (II) > (I) > (IV)

(4) (II) > (III) > (IV) > (I)

NTA Ans. (3)

13. The solubility product of $Cr(OH)_3$ at 298 K is 6.0×10^{-31}. The concentration of hydroxide ions in a saturated solution of $Cr(OH)_3$ will be :

(1) $(18 \times 10^{-31})^{1/4}$ (2) $(2.22 \times 10^{-31})^{1/4}$

(3) $(4.86 \times 10^{-29})^{1/4}$ (4) $(18 \times 10^{-31})^{1/2}$

NTA Ans. (1)

Sol. $Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq.) + 3OH^-(aq.)$

(s) (3s)

$k_{sp} = 27(s)^4 = 6 \times 10^{-31}$

$\Rightarrow [3(s)]^4 = 18 \times 10^{-31}$

$[OH^-] = 3(s) = [18 \times 10^{-31}]^{1/4}$

14. 5 g of zinc is treated separately with an excess of

(a) dilute hydrochloric acid and

(b) aqueous sodium hydroxide.

The ratio of the volumes of H_2 evolved in these two reactions is :

(1) 1 : 4 (2) 1 : 2 (3) 2 : 1 (4) 1 : 1

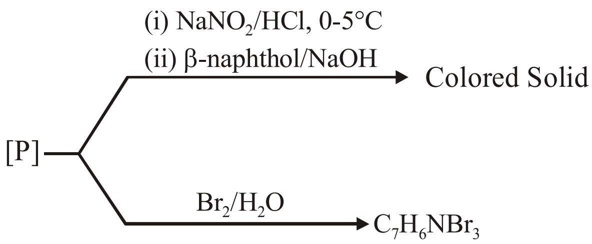
NTA Ans. (4)

Sol. $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

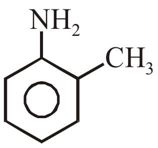
$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

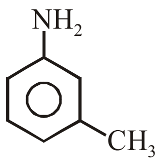
The ratio of the volume of H_2 is 1 : 1

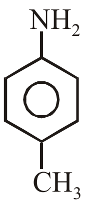
15. Consider the following reactions,

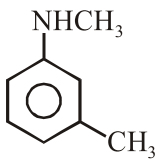


The compound [P] is :

(1) 

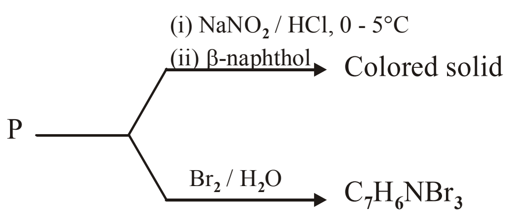
(2) 

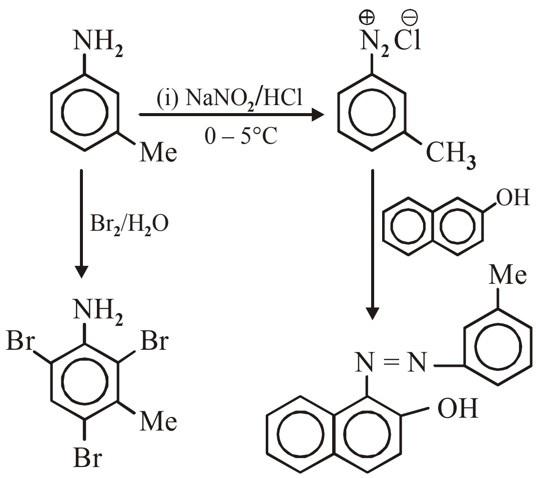
(3) 

(4) 

NTA Ans. (2)

Sol.





(colored dye)

16. A, B and C are three biomolecules. The results of the tests performed on them are given below:

	Molisch's Test	Barfoed Test	Biuret Test
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

A, B and C are respectively :

(1) A = Glucose, B = Fructose, C = Albumin

(2) A = Lactose, B = Fructose, C = Alanine

(3) A = Lactose, B = Glucose, C = Alanine

(4) A = Lactose, B = Glucose, C = Albumin

NTA Ans. (4)

Sol. Alanine does not show **Biuret test** because **Biuret test** is used for deduction of peptide linkage & alanine is amino acid.

Albumine is protein so have paptide linkage so it gives positive **Biuret test**.

Positive **Barfoed test** is shown by monosaccharide but not disaccharide. Positive **Molisch's test** is shown by glucose.

17. The reaction of $H_3N_3B_3Cl_3$ (A) with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Further, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively, are:

(1) Boron nitride and MeBr

(2) Borazine and MeMgBr

(3) Borazine and MeBr

(4) Diborane and MeMgBr

NTA Ans. (2)

Sol.

$$\underset{(A)}{H_3N_3B_3Cl_3} + 3LiBH_4 \xrightarrow[\text{(T.H.F.)}]{\text{In tetrahydrofurane}} \underset{\substack{(B)\\ \text{Inorganic Benzene}\\ \text{(Borazene)}}}{H_3N_3B_3H_3} + 3LiCl + 3BH_3.THF$$

$$\underset{(A)}{H_3N_3B_3Cl_3} + 3CH_3MgBr \xrightarrow{} \underset{(C)}{H_3N_3B_3(CH_3)_3} + 3MgBrCl$$

18. The isomer(s) of $[Co(NH_3)_4Cl_2]$ that has/have a Cl–Co–Cl angle of 90°, is/are :

(1) meridional and trans

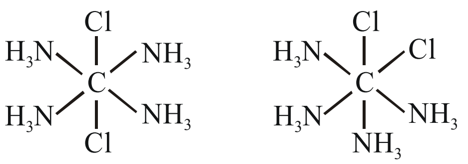
(2) cis and trans

(3) trans only

(4) cis only

NTA Ans. (4)

Sol. $\left[Co(NH_3)_4Cl_2\right]$ has 2 geometrical isomers



trans cis

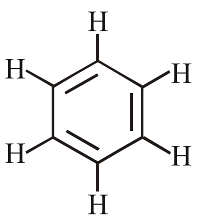
cis isomer has Cl–Co–Cl angle of 90°

19. The number of sp^2 hybrid orbitals in a molecule of benzene is :

(1) 24 (2) 6 (3) 12 (4) 18

NTA Ans. (4)

Sol.



Each carbon atom is sp^2 hybridized

Therefore each carbon has 3 sp^2 hybrid orbitals.

Hence total sp^2 hybrid orbitals are 18.

20. The true statement amongst the following is:

(1) Both ΔS and S are functions of temperature.

(2) S is not a function of temperature but ΔS is a function of temperature.

(3) Both S and ΔS are not functions of temperature.

(4) S is a function of temperature but ΔS is not a function of temperature.

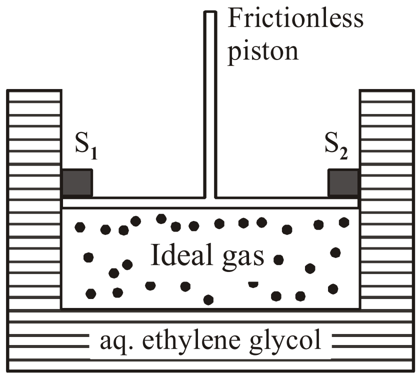
NTA Ans. (1)

Sol. $ds = \int \frac{q_{rev.}}{T}$

21. A cylinder containing an ideal gas (0.1 mol of 1.0 dm^3) is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at its freezing point. If the stoppers S_1 and S_2 (as shown in the figure) are suddenly withdrawn, the volume of the gas in litres after equilibrium is achieved will be____.

(Given, K_f (water) = 2.0 K kg mol^{-1},

R = 0.08 dm^3 atm K^{-1} mol^{-1})



NTA Ans. (2.18 to 2.23)

Sol. $0 - T_f' = 2 \times 0.5 = 1$

$T_f' = -1°C = 272\ K$

for gas $P = \frac{0.1 \times 0.08 \times 272}{1}$

P = 2.176 atm

$P_1V_1 = P_2V_2$

$2.176 \times 1 = 1 \times V_2$

V_2 = 2.176 litre

22. 10.30 mg of O_2 is dissolved into a liter of sea water of density 1.03 g/mL. The concentration of O_2 in ppm is__________.

NTA Ans. (10)

Sol. ppm = $\frac{10.3 \times 10^{-3}}{1030} \times 10^6 = 10$

23. A sample of milk splits after 60 min. at 300 K and after 40 min. at 400 K when the population of *lactobacillus acidophilus* in it doubles. The activa tion energy (in kJ/ mol) for this process is closest to__________.

(Given, R = 8.3 J mol^{-1} K^{-1}, $ln\left(\frac{2}{3}\right)$ = 0.4, e^{-3} = 4.0)

NTA Ans. (3.98 to 3.99 or –3.98 to –3.99)

Sol. $\ln\left(\frac{t_1}{t_2}\right) = \frac{-Ea}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$

$\ln\left(\frac{60}{40}\right) = \frac{-Ea}{8.3}\left[\frac{1}{400} - \frac{1}{300}\right]$

E = 0.4 × 1200 × 8.3

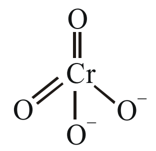
= 3.984 kJ/mole

24. The sum of the total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is ____________.

NTA Ans. (12)

Sol.

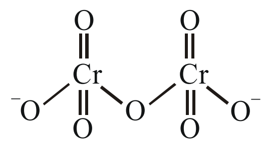
Chromate CrO_4^{2-}



Total Cr-O bonds = 6

(4σ + 2π)

Dichromate $Cr_2O_7^{2-}$



Total Cr-O bonds = 12

(8σ + 4π)

Total number of bonds between chromium and oxygen in both structures are 18.

Note :- But answer of NTA is 12. They consider only linkages between Chromium and Oxygen but in question total no. of bonds are asked so σ and π bonds must be considered separately.

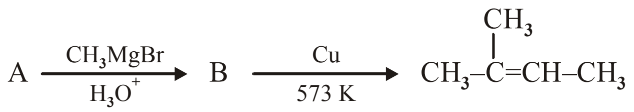
25. Consider the following reactions

$A \xrightarrow[(ii)H_3O^+]{(i)CH_3MgBr} B \xrightarrow[573K]{Cu}$ 2-methyl 2-butene

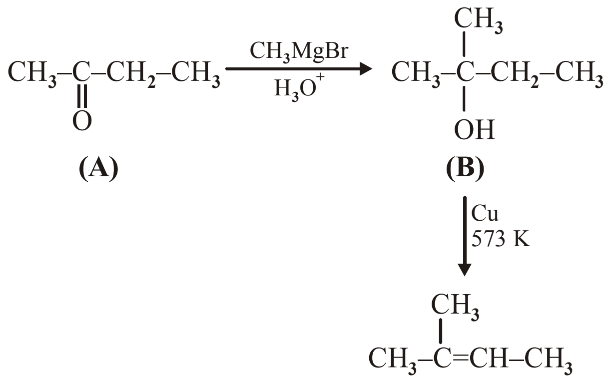
The mass percentage of carbon in A is ______.

NTA Ans. (66.66 to 66.67)

Sol.



(2-methyl-2-butene)



(2-methyl-2-butene)

$C \Rightarrow 12 \times 4 = 48$

$H \Rightarrow 8 \times 1 = 8$

$O \Rightarrow 16 \times 1 = 16$

————————

Total 72

$$\% \text{ of } C = \frac{48}{72} \times 100 = 66.66\%$$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

MATHEMATICS — TEST PAPER WITH ANSWER & SOLUTION

1. Let [t] denote the greatest integer ≤ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

(1) $\sqrt{A+5}$ (2) $\sqrt{A+1}$

(3) $\sqrt{A}$ (4) $\sqrt{A+21}$

NTA Ans. (2)

Sol. $A = \lim_{x\to 0} x\left[\frac{4}{x}\right] = \lim_{x\to 0} x\left(\frac{4}{x}\right) - x\left\{\frac{4}{x}\right\} = 4$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore\ x = \sqrt{A+1}$ i.e. $\sqrt{5}$

2. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) infinitely many solutions, (x, y, z) satisfying x = 2z

(2) no solution

(3) only the trivial solution

(4) infinitely many solutions, (x, y, z) satisfying y = 2z

NTA Ans. (1)

Sol. $7x + 6y - 2z = 0$ .... (1)

$3x + 4y + 2z = 0$ .... (2)

$x - 2y - 6z = 0$ .... (3)

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0 \Rightarrow \text{infinite solutions}$$

Now (1) + (2) ⇒ y = –x put in (1), (2) & (3) all will lead to x = 2z

3. If x = 2sinθ – sin2θ and y = 2cosθ – cos2θ, θ ∈ [0, 2π], then $\frac{d^2y}{dx^2}$ at θ = π is :

(1) $\frac{3}{2}$ (2) $-\frac{3}{4}$

(3) $\frac{3}{4}$ (4) $-\frac{3}{8}$

NTA Ans. (4)

Sol. x = 2sinθ – sin2θ

$\Rightarrow \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta = 4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)$

y = 2cosθ – cos2θ

$\Rightarrow \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta = 4\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$

$$\Rightarrow \frac{dy}{dx} = \cot\left(\frac{3\theta}{2}\right) \Rightarrow \frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)}{4\sin\left(\frac{\theta}{2}\right)\sin\frac{3\theta}{2}}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = \frac{3}{8}$$

Alternate :-

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin\theta + 2\sin 2\theta}{2\cos\theta - 2\cos 2\theta} = \frac{\sin\theta - \sin 2\theta}{-\cos\theta + \cos 2\theta}$$

$$\frac{d^2y}{dx^2}\cdot\frac{dx}{d\theta} = \frac{(-\cos\theta + \cos 2\theta)(\cos\theta - 2\cos 2\theta) - (\sin\theta - \sin 2\theta)(\sin\theta - 2\sin 2\theta)}{(-\cos\theta + \cos 2\theta)^2}$$

$$\frac{d^2y}{dx^2}.(-2-2) = \frac{(+1+1)(-1-2)-(0)}{(1+1)^2}$$

$$\frac{d^2y}{dx^2}(-4) = \frac{2\times -3}{4} = -\frac{3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{8}$$

Answer should be $\frac{3}{8}$. No options is correct.

4. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is :

(1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{5}{3}}$

NTA Ans. (2)

Sol. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $a > b$;

$$2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}} \Rightarrow b^2=\frac{4}{3}$$

tangent $y=\frac{-x}{6}+\frac{4}{3}$ compare with

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$\Rightarrow m=\frac{-1}{6}\Rightarrow\sqrt{\frac{a^2}{36}+\frac{4}{3}}=\frac{4}{3} \Rightarrow a = 4;$$

$$e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{2}\sqrt{\frac{11}{3}}$$

5. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

(1) 26 (2) 25 (3) 28 (4) 24

NTA Ans. (2)

Sol. $ax^2 - 2bx + 5 = 0 < \begin{matrix}\alpha \\ \alpha\end{matrix}$

$$\Rightarrow \alpha=\frac{b}{a};\ \alpha^2=\frac{5}{a} \Rightarrow b^2 = 5a$$

$$x^2 - 2bx - 10 = 0 < \begin{matrix}\alpha \\ \beta\end{matrix} \Rightarrow \alpha^2 - 2b\alpha - 10 = 0$$

$$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20;\ \alpha\beta = -10 \Rightarrow \beta^2 = 5$$

$$\Rightarrow \alpha^2 + \beta^2 = 25$$

6. Given : $f(x) = \begin{cases} x & , 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \le 1 \end{cases}$

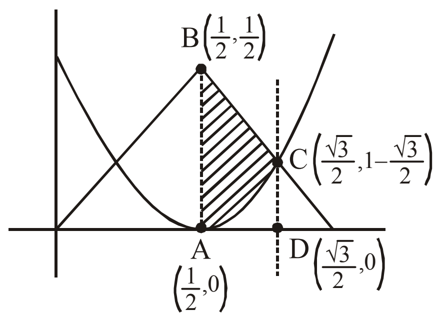
and $g(x) = \left(x-\frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

(1) $\frac{1}{3}+\frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{4}-\frac{1}{3}$

(3) $\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{1}{2}-\frac{\sqrt{3}}{4}$

NTA Ans. (2)

Sol.



Required area = Area of trepezium ABCD –

Area of parabola between $x = \frac{1}{2}$ & $x=\frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4}-\frac{1}{3}$$

7. A random variable X has the following probability distribution :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then $P(X > 2)$ is equal to :

(1) $\frac{7}{12}$ (2) $\frac{23}{36}$ (3) $\frac{1}{36}$ (4) $\frac{1}{6}$

NTA Ans. (2)

Sol. $\sum P(X)=1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = -1$ (rejected) $\Rightarrow K = \frac{1}{6}$

$P(X > 2) = K + 2K + 5K^2 = \frac{23}{36}$

8. If $x=\sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta$ and $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$, for $0<\theta<\frac{\pi}{4}$, then :

(1) $y(1 + x) = 1$ (2) $x(1 + y) = 1$

(3) $y(1 - x) = 1$ (4) $x(1 - y) = 1$

NTA Ans. (3)

Sol. $x=\sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta = 1 - \tan^2\theta + \tan^4\theta + ...$

$\Rightarrow x = \cos^2\theta$

$y=\sum_{n=0}^{\infty}\cos^{2n}\theta \Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + ...$

$\Rightarrow y=\frac{1}{\sin^2\theta} \Rightarrow y=\frac{1}{1-x}$

$\Rightarrow y(1 - x) = 1$

9. Let a function $f : [0, 5] \to \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as :

$F(x)=\int_1^x t^2 g(t)\,dt$, where $g(t)=\int_1^t f(u)\,du$.

Then for the function F, the point $x = 1$ is :

(1) a point of local minima.

(2) not a critical point.

(3) a point of inflection.

(4) a point of local maxima.

NTA Ans. (1)

Sol. $F'(x) = x^2 g(x) = x^2\int_1^x f(u)\,du \Rightarrow F'(1) = 0$

$F''(x) = x^2 f(x) - 2x\int_1^x f(u)\,du$

$F''(1) = 1.f(1) - 2 \times 0$

$F''(1) = 3$

$F'(1) = 0$ and $F''(1) = 3 > 0$ So, Minima

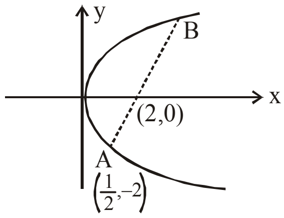
10. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2},-2\right)$, then the equation of the tangent to it at B is :

(1) $2x + y - 24 = 0$ (2) $x - 2y + 8 = 0$

(3) $2x - y - 24 = 0$ (4) $x + 2y + 8 = 0$

NTA Ans. (2)

Sol. $y^2 = 8x$



$4t_1 = -2 \Rightarrow t_1 = -\frac{1}{2}$,

$t_1.t_2 = -1$

$t_2 = -\frac{1}{t_1}$

$\Rightarrow t_2 = 2$

So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

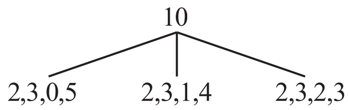
$8y = 4(x + 8) \Rightarrow 2y = x + 8$

11. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

(1) $\frac{945}{2^{11}}$ (2) $\frac{965}{2^{11}}$ (3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{10}}$

NTA Ans. (3)

Sol. 10 different balls in 4 different boxes.



$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times3!\times0!\times5!}+4!\times\frac{10!}{2!\times3!\times1!\times4!}+4!\times\frac{10!}{(2!)^2\times2!\times(3!)^2\times2!}\right)$

$=\frac{17\times945}{2^{15}}$

12. If $A=\{x\in\mathbf{R}:|x|<2\}$ and $B=\{x\in\mathbf{R}:|x-2|\geq3\}$; then :

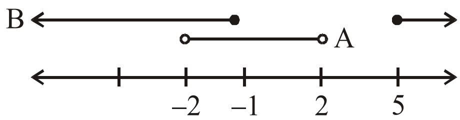
(1) $A\cup B=\mathbf{R}-(2, 5)$ (2) $A\cap B=(-2, -1)$

(3) $B - A = \mathbf{R} - (-2, 5)$ (4) $A - B = [-1, 2)$

NTA Ans. (3)

Sol. $A : x \in (-2, 2)$; $B : x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



13. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is :

(1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$ (3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$

NTA Ans. (4)

Sol. $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Let y = vx

$$\frac{dy}{dx}=v+x.\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{xvx}{x^2+v^2x^2}=\frac{v}{1+v^2}$$

$$x\frac{dv}{dx}=\frac{v}{1+v^2}-v=\frac{v-v-v^3}{1+v^2}=-\frac{v^3}{1+v^2}$$

$$\int\frac{1+v^2}{v^3}.dv=\int-\frac{dx}{x}$$

$$\Rightarrow \int v^{-3}.dv+\int\frac{1}{v}dv=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{v^{-2}}{-2}+\ell n v=-\ell n x+\lambda$$

$$\Rightarrow -\frac{1}{2v^2}+\ell n\left(\frac{y}{x}\right)=-\ell n x+\lambda$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2}+\ell n y-\ell n x=-\ell n x+\lambda$$

$$\Rightarrow -\frac{1}{2}+0=\lambda \Rightarrow \lambda=-\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2}+\ell n y+\frac{1}{2}=0 \text{ at } y = e$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2}+1+\frac{1}{2}=0 \Rightarrow \frac{x^2}{2e^2}=\frac{3}{2}\Rightarrow x^2=3e^2$$

$$\therefore x=\sqrt{3}e$$

14. If $\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}=\lambda\tan\theta+2\log_e|f(\theta)|+C$

where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

(1) (–1, 1 + tanθ) (2) (–1, 1 – tanθ)

(3) (1, 1 – tanθ) (4) (1, 1 + tanθ)

NTA Ans. (1)

Sol. $I=\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}$

$$=\int\frac{\sec^2\theta\, d\theta}{\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}}=\int\frac{(1-\tan^2\theta)\sec^2\theta\, d\theta}{(1+\tan\theta)^2}$$

$\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$$I=\int\frac{1-t^2}{(1+t)^2}dt=\int\frac{(1-t)(1+t)}{(1+t)^2}dt$$

$$=\int\frac{1}{1+t}-\frac{t}{1+t}dt$$

$$=\ell n|1+t|-\int\left(\frac{1+t}{1+t}-\frac{1}{1+t}\right)dt$$

$= \ell n|1 + t| – t + \ell n|1 + t|$

$= 2\ell n|1 + t| – t + C$

$= 2\ell n|1 + \tan\theta| – \tan\theta + C$

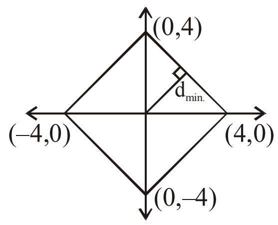
$\lambda = –1, f(\theta) = 1 + \tan\theta$

15. If z be a complex number satisfying |Re(z)| + |Im(z)| = 4, then |z| cannot be

(1) $\sqrt{\frac{17}{2}}$ (2) $\sqrt{10}$ (3) $\sqrt{8}$ (4) $\sqrt{7}$

NTA Ans. (4)

Sol.



z = x + iy |x| + |y| = 4

$|z|=\sqrt{x^2+y^2}\Rightarrow|z|_{min}=\sqrt{8}\,\&\,|z|_{max}=4=\sqrt{16}$

So |z| cannot be $\sqrt{7}$

16. If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, T (2) T, T (3) F, F (4) T, F

NTA Ans. (2)

Sol. $p \to (p \wedge \sim q)$ is F $\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T

$\therefore$ p is T, q is T

17. Let a – 2b + c = 1.

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then :

(1) $f(-50) = 501$ (2) $f(-50) = -1$

(3) $f(50) = 1$ (4) $f(50) = -501$

NTA Ans. (3)

Sol. $R_1 \to R_1 + R_3 - 2R_2$

$$f(x)=\begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a + c - 2b)\left((x + 3)^2 - (x + 2)(x + 4)\right)$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

18. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :

(1) 1 : 8 (2) 1 : 16

(3) 8 : 1 (4) 16 : 1

NTA Ans. (4)

Sol. $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

$= {}^{16}C_r\,(x)^{16-2r}\times\frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$\Rightarrow T_9 = {}^{16}C_8\frac{1}{\cos^8\theta\sin^8\theta}$

$= {}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$

for $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1=\frac{\pi}{4}$

for $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2=\frac{\pi}{8}$

$\frac{\ell_2}{\ell_1}=\frac{(\sin 2\theta_1)^8}{(\sin 2\theta_2)^8}=\left(\sqrt{2}\right)^8=\frac{16}{1}$

19. Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1}=200$ and $\sum_{n=1}^{100} a_{2n}=100$, then $\sum_{n=1}^{200} a_n$ is equal to :

(1) 225 (2) 175 (3) 300 (4) 150

NTA Ans. (4)

Sol. $\sum_{n=1}^{100} a_{2n+1}=200 \Rightarrow a_3 + a_5 + a_7 + ... + a_{201} = 200$

$\Rightarrow ar^2\frac{(r^{200}-1)}{(r^2-1)}=200$

$\sum_{n=1}^{100} a_{2n}=100 \Rightarrow a_2 + a_4 + a_6 + ... + a_{200} = 100$

$\Rightarrow \frac{ar(r^{200}-1)}{(r^2-1)}=100$

On dividing r = 2

on adding $a_2 + a_3 + a_4 + a_5 + ... + a_{200} + a_{201} = 300$

$\Rightarrow r(a_1 + a_2 + a_3 + ... + a_{200}) = 300$

$\Rightarrow \sum_{n=1}^{200} a_n=150$

20. Let f and g be differentiable functions on **R** such that fog is the identity function. If for some a, b $\in$ **R**, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

(1) $\frac{2}{5}$ (2) 1 (3) $\frac{1}{5}$ (4) 5

NTA Ans. (3)

Sol. $f(g(x)) = x$

$f'(g(x))\ g'(x) = 1$

put $x = a$

$\Rightarrow f'(b)\ g'(a) = 1$

$f'(b) = \frac{1}{5}$

21. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ______.

NTA Ans. (14)

Sol. Common term are : 23, 51, 79, ... T_n

$T_n \le 407 \quad \Rightarrow 23 + (n-1)28 \le 407$

$\Rightarrow n \le 14.71$

$n = 14$

22. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b}\cdot\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $\left|\vec{a}\times(\vec{b}\times\vec{c})\right|$ is equal to _____.

NTA Ans. (30)

Sol. $\vec{b}\cdot\vec{c} = 10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3} = 10 \Rightarrow |\vec{c}| = 4$

$\left|\vec{a}\times(\vec{b}\times\vec{c})\right| = |\vec{a}|\,|\vec{b}\times\vec{c}|$

$= \sqrt{3}\,.\,5\,.\,4\,.\,\sin\frac{\pi}{4} = 30$

23. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\frac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \ne -7$, then planes will be intersecting & distance between them will be 0

24. If $C_r \equiv {}^{25}C_r$ and

$C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to _____.

NTA Ans. (51)

Sol. $S = 1.{}^{25}C_0 + 5.{}^{25}C_1 + 9.{}^{25}C_2 + ... + (101){}^{25}C_{25}$

$S = 101{}^{25}C_{25} + 97{}^{25}C_1 + ...\ + 1{}^{25}C_{25}$

$2S = (102)\ (2^{25})$

$S = 51\ (2^{25})$

25. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ______.

NTA Ans. (36)

Sol. Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for I[st] circle

$\Rightarrow \frac{|-18-8+k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \quad \Rightarrow k_{max} = 36$